Temperature & Level control in a stirred tank heater.

material balance

$$\frac{d(Ah\rho)}{dt} = F_i\rho - F\rho.$$

ρ const; A = const.

F = volumetric flowrate

$$A \frac{dh}{dt} = F_i - F.$$

$$\frac{d(\rho C_p A h T)}{dt} = F_i \rho C_p T_i - F\rho C_p T + Q$$

ρC_p = const.

$$Ah\frac{dT}{dt} + AT\frac{dh}{dt} = F_i T_i - FT + \frac{Q}{\rho C_p}.$$

$$Ah\frac{dT}{dt} + TF_i - \cancel{TF} = F_i T_i - \cancel{FT} + \frac{Q}{\rho C_p}$$

$$Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho C_p}.$$

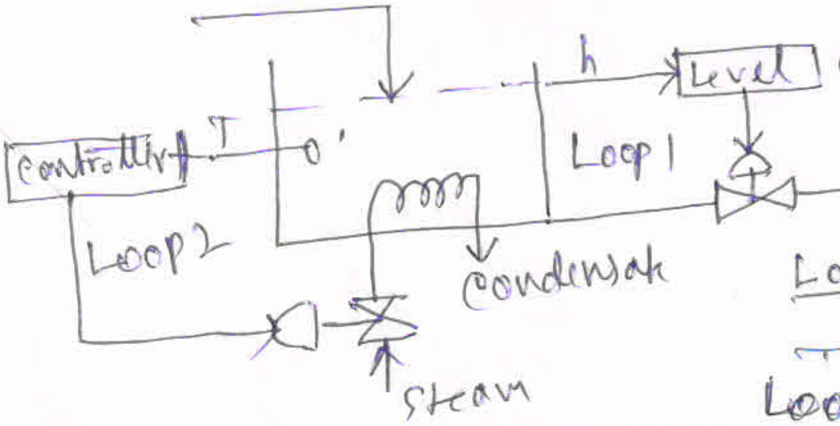
no. of variables h, T, F_i, F, T_i, Q. 6.

no. of eq. 2.

dOF = 4, T_i, F_i are specified by external world.

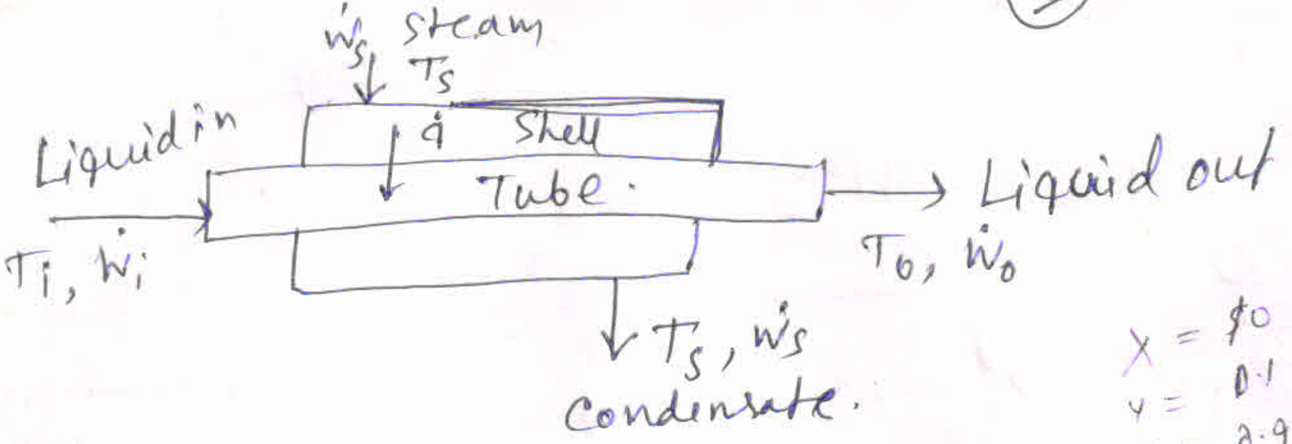
∴ dOF = 4 − 2 = 2. 2 control objectives.

2 → feed back loops are required to control h, T.



	control variables	manipulated	measured
Loop 1 →	h	F	h
Loop 2	T	Q	T

Heat exchanger control (V)



x = 10
y = 0.1
z = 0.9

$W_i = W_0 = \dot{W}$.

$\dot{q} = UA(T_s - T_0)$. Tube side.

$$W_t C_p \frac{dT_0}{dt} = \dot{W} C_p (T_i - T_0) + UA(T_s - T_0).$$

variables $T_0, T_i, T_s, \dot{W}, \dot{W}_s$ = (4)

Shell side

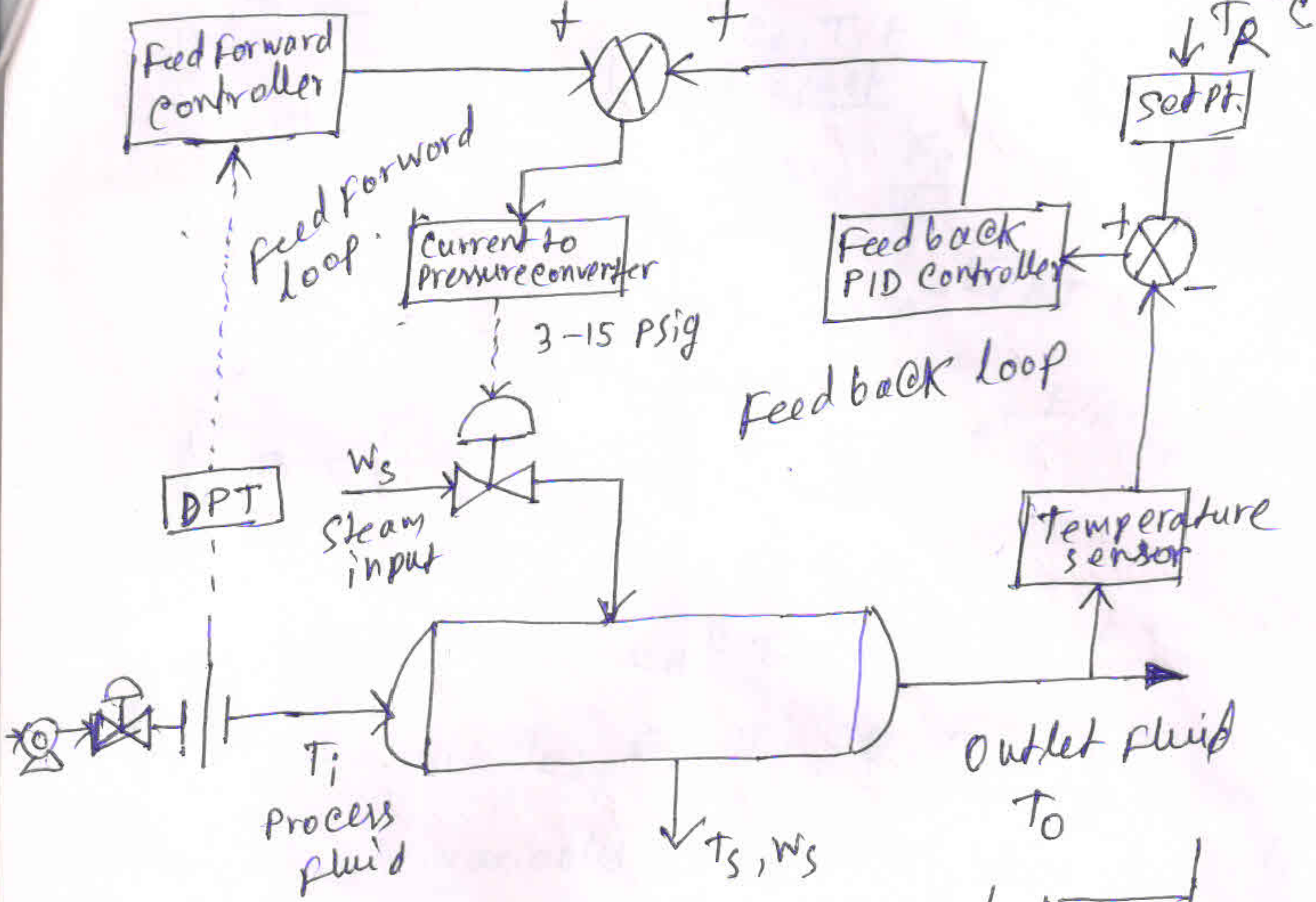
$W_s C_p^s \frac{dT_s}{dt} =$

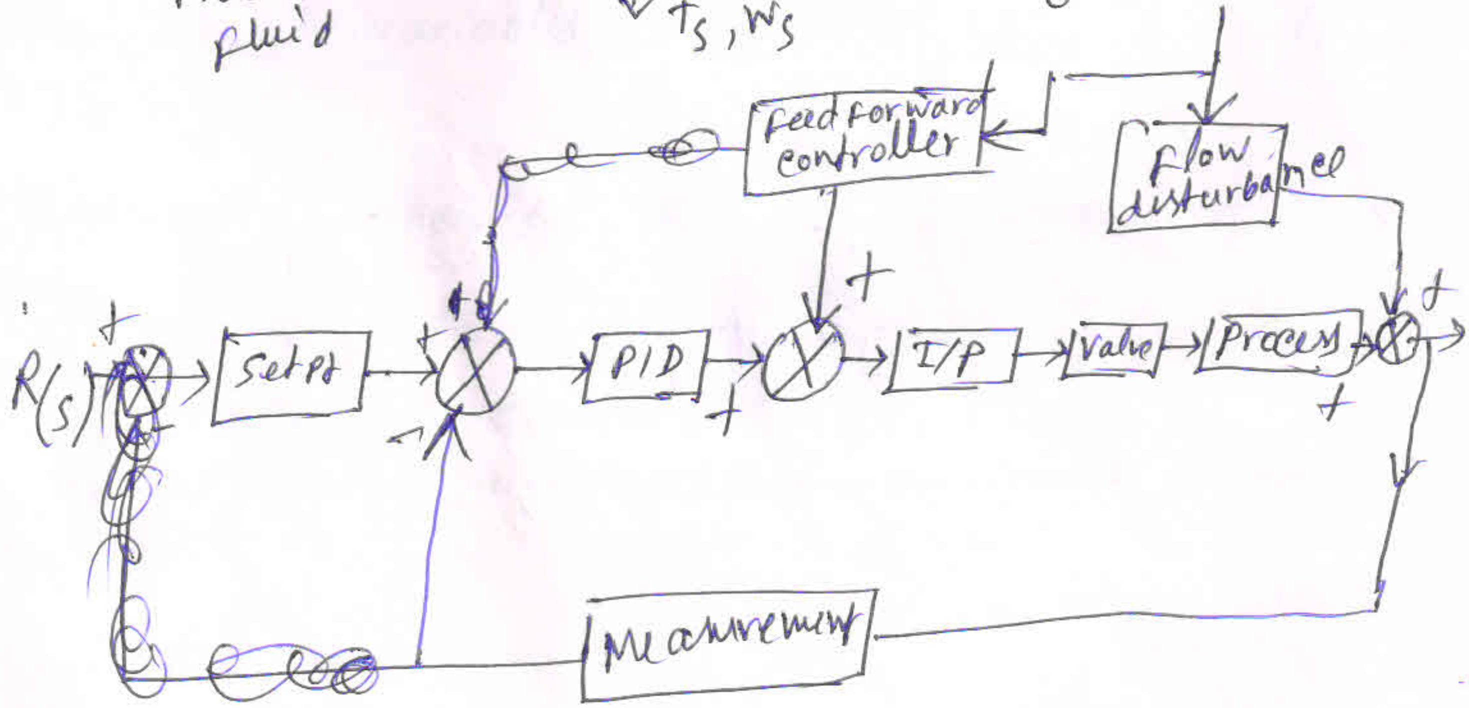
$W_s \frac{dh_s}{dt} =$

Here $f = 4 - 1 = 3$
specified by the externernal world is T_s ⇒ specified variable no. (1), T_i = (1)

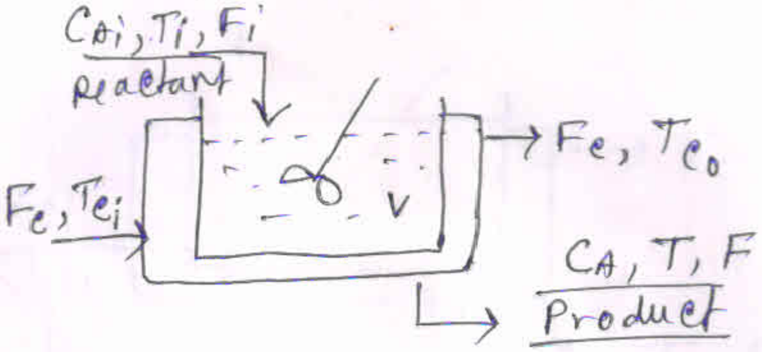
∴ control objective = 3 - 2 = 1

	controlled variable	Mainipulate variable
1st loop Feed back	T_0 measured variable.	W_s.
2nd loop Feed forward	T_0 measured variable is $\dot{W}$.	W_s


Feed forward controller
Feed forword loop
Current to pressure converter
3-15 Psig
Feed back PID Controller
Feed back Loop
Set Pt.
T_R °C
DPT
W_s
Steam input
Temperature sensor
T_i
Process fluid
T_s, W_s
Outlet fluid
T_O


Feed forward controller
Flow disturbance
R(s)
Set Pt
PID
I/P
Valve
Process
Measurement

Control of CSTR



$A \rightarrow B$ + Heat
exothermic reaction.

$$\frac{dV}{dt} = F_i - F \; ; \quad \frac{dT_{co}}{dt} = \frac{F_c}{V_c}(T_{ci} - T_{co}) + \frac{Q}{\rho_c C_{pc} V_c}$$

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - k_0 e^{-E/RT} C_A$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + J k_0 e^{-E/RT} C_A - \frac{Q}{\rho C_p V}$$

$$J = (-\Delta H_r)/\rho C_p$$

State variables V, C_A, T ⇒ output variables.
input " C_{Ai}, F_i, T_i, Q, F.

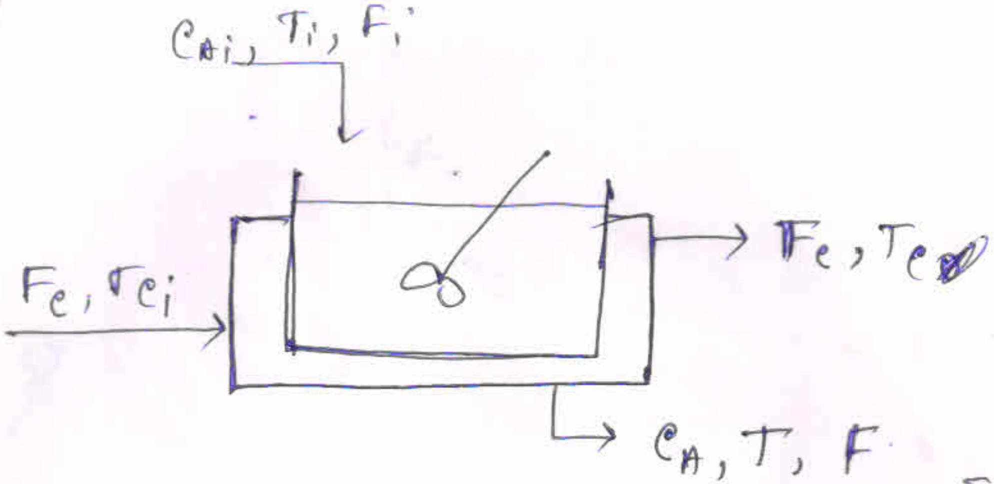
Total no. of variables $V, T, T_i, T_{ci}, T_{co}, F, F_i, F_c, C_A, C_{Ai} = 10$

Externally specified $F_i, C_{Ai}, T_i, F_c, T_{ci} = 5$.

∴ unspecified variable $= 10 - 5 = 5$.

∴ Modeling equation $= 3$.

∴ Control objective / no. of controlled or manipulate variables $= 5 - 3 = 2$.



$(F_i = F)$

$$\frac{dC_A}{dt} = \frac{F_i}{V}(C_{Ai} - C_A) - k_0 e^{-E/RT} C_A .$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + J k_0 e^{-E/RT} C_A - \frac{UA(T - T_c)}{\rho c_{pc} V}$$

$$\frac{dT_c}{dt} = \frac{F_c}{V_c}(T_{ci} - T_c) + \frac{(T - T_c) UA}{\rho c_{pc} V}$$

Total no. of variable $C_A, T, T_c, T_i, C_{Ai}, T_{ci}, F_i, F_c, V, V_c$ = 10 no.

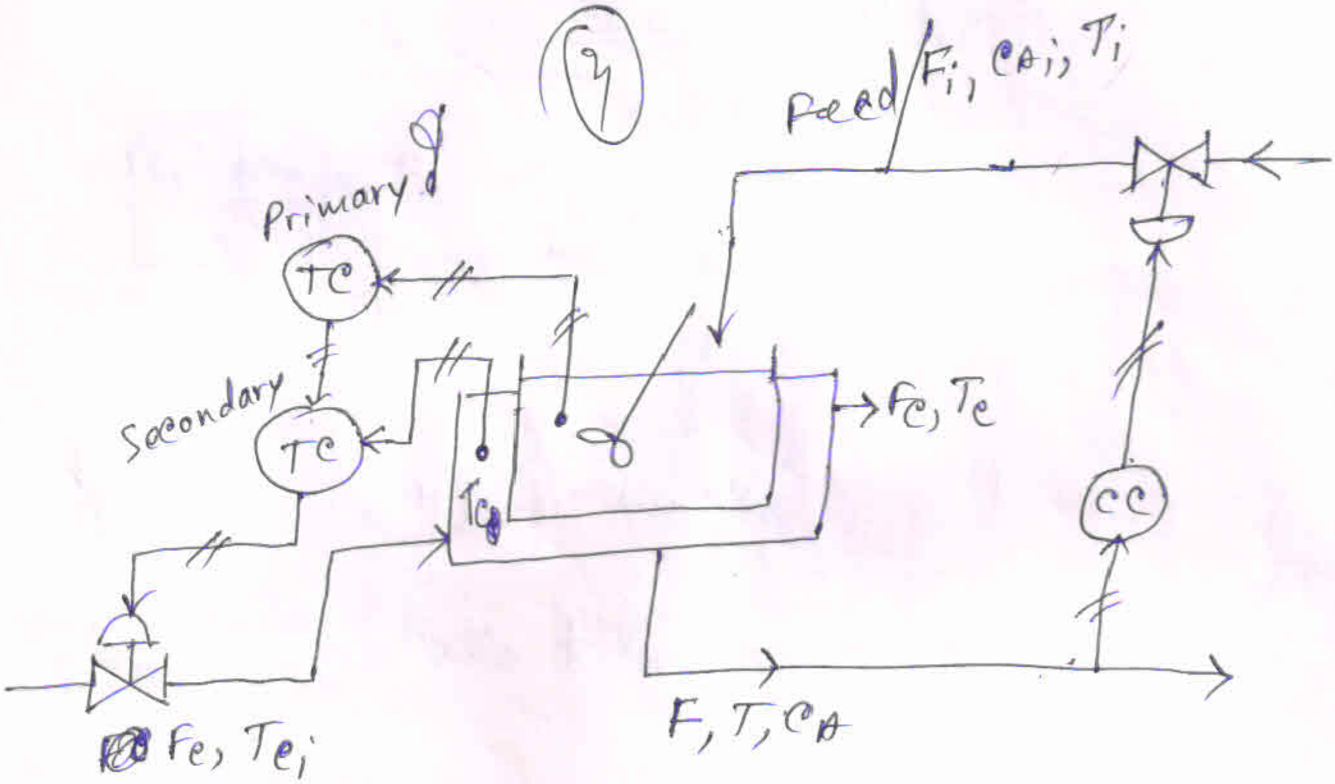
no. of equation = 3 ;
no. of specified variables = $V, V_c, T_i, C_{Ai}, T_{ci}$ = 5 .
∴ no. of controlled objective = 10 − (5+3) = 2 . ✓

Now V, and V_c and be specified so level of tank and jacket volume are constant.

(3)

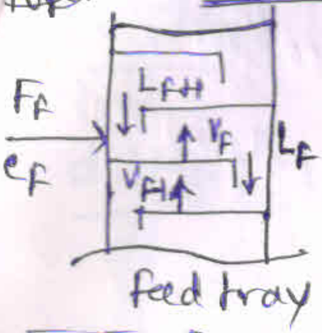
	c_A control by	T control by
1)	F_i	F_c (or T_{co}).
2)	F_c	F_i
3) ✓	F_i	F_c (with T_c as a secondary measurement in cascade control).

(4)



Control of Distillation column (Ideal Binary mixture)

Feed Tray $(i=f)$

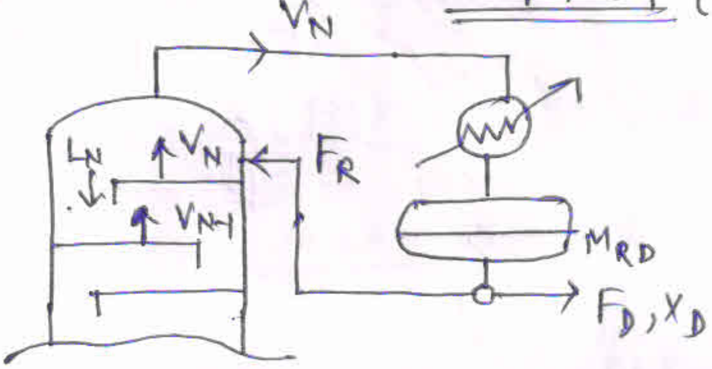


$$\frac{d(M_f)}{dt} = F_f + L_{f+1} + V_{f-1} - L_f - V_f$$

$$\frac{d(M_f x_f)}{dt} = F_f c_f + L_{f+1}x_{f+1} + V_{f-1}y_{f-1} - L_f x_f - V_f y_f$$

No of equation = 2

Top Tray $(i=N)$.



$$\frac{d(M_N)}{dt} = F_R + V_{N-1} - L_N - V_N$$

$$\frac{d(M_N x_N)}{dt} = F_R x_D + V_{N-1}y_{N-1} - L_N x_N - V_N y_N$$

No of equation = 2

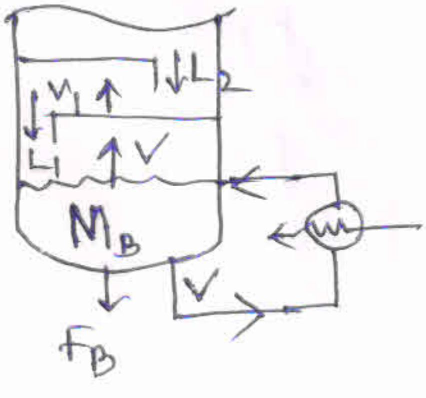
Reflux drum

$$\frac{d(M_{RD})}{dt} = V_N - F_R - F_D$$

$$\frac{d(M_{RD}x_D)}{dt} = V_N y_N - (F_R + F_D)x_D$$

No of equation = 2

Bottom Tray $i=1$



$$\frac{d(M_1)}{dt} = L_2 + V - L_1 - V_1 = L_2 - L_1$$

$$\frac{d(M_1 x_1)}{dt} = L_2 x_2 + V y_B - L_1 x_1 - V_1 y_1$$

No of equation = 2

~~refluxdrum~~ Column base

$$\frac{d(M_B)}{dt} = L_1 - V - F_B$$
$$\frac{d(M_B X_B)}{dt} = L_1 X_1 - V Y_B - F_B X_B$$

No of equation $= 2$

ith tray ($i = 2, \ldots N-1$ and $i \neq F$) Total $(N-3)$ Tray

$$\frac{d(M_i)}{dt} = L_{i+1} - L_i + V_{i-1} - V_i = L_{i+1} - L_i$$
$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} Y_{i-1} - L_i x_i - V_i Y_i$$

No of equations $= 2(N-3)$

Equilibrium Relation Ship

$$Y_i = \frac{\alpha x_i}{1+(\alpha-1)x_i} \quad i = 1, \ldots, N+1.$$

∴ No of equation $= N+1$.

Hydraulic equation:

$L_i = F(M_i) \quad i = 1, 2, \ldots, N$ } No of equation $= N$.

To neglect momentum balance equation in each tray.

∴ Equilibrium relation ship	$N+1$.
Hyraulic equ.	$1N$.
Feed tray	2.
Top + Bottom tray	4
Refluxdrum + Column base	4
Midde Trayes	$2(N-3)$.
Total.	$= 4N+5$.

No. of variables

⇒ N+2	X_i	i=1, 2, ---, N, D, B. (Liquid composition)
⇒ N+1	Y_i	i=1, 2 --- N, B (vapor ″)
⇒ N	L_i	i=1, 2 --- N.
⇒ N+2	M_i	Liquid holdups..
⇒ 6		F_F, C_F, F_D, F_B, F_R, V

Total = 4N + 11.

∴ Degrees of freedom = 4N+11 − 4N−5
= 6.

Specification of disturbance variables:

C_F, & F_F are specified by the external world.

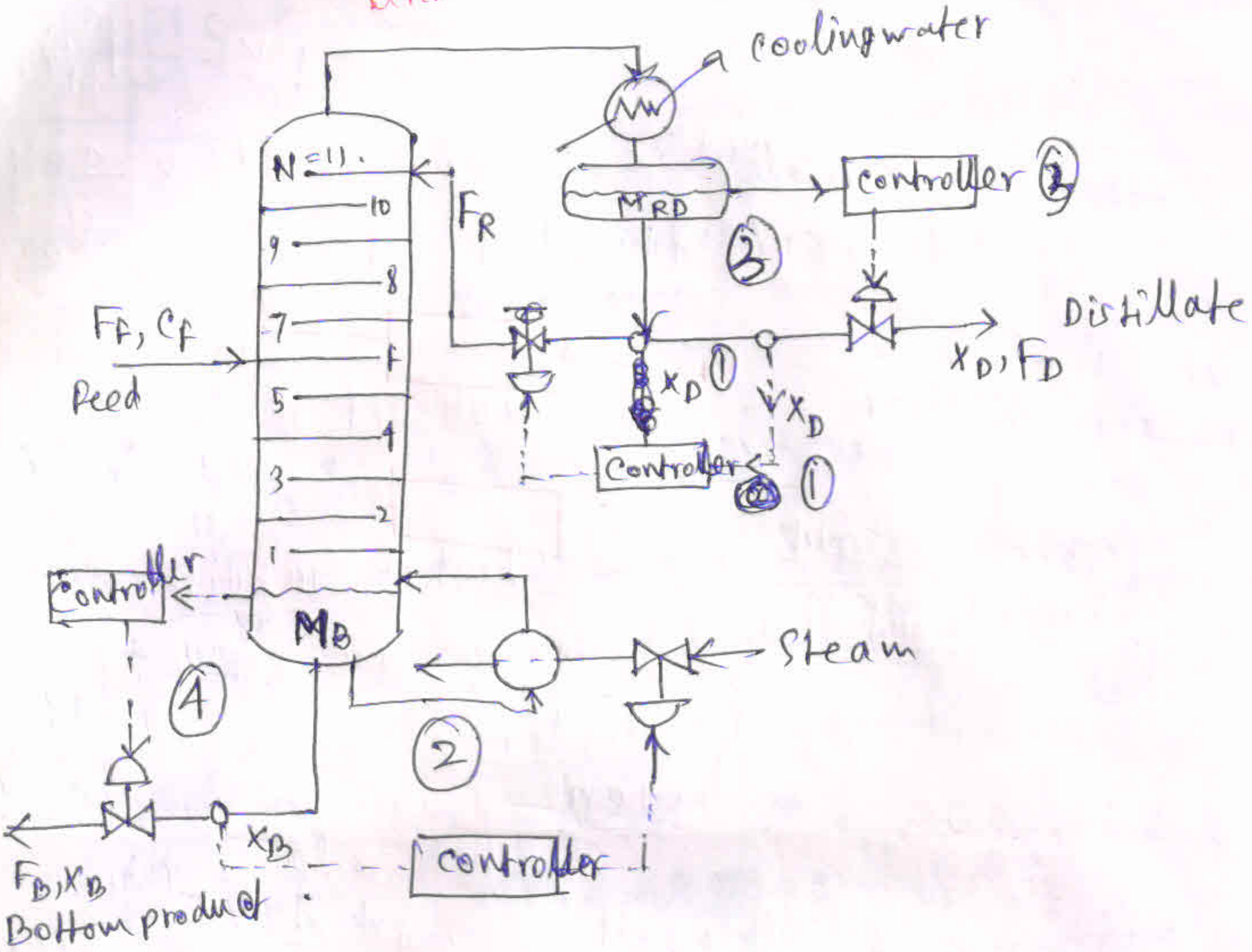
Specification of control objective:

6−2 = 4 remains unspecified.

∴ control objective = 4.

1> X_D 2> X_B, 3> M_{RD}, 4> M_B.

$2N+7-(2N+3) =$ variable unspecified eq



Loop (1) →	controlled variable	Manipulated variable
Composition controller →	x_D	F_R
Loop (2)		
composition controller →	x_B	Steam flowrate / Heating rate
Loop (3)		
Level controller →	M_{RD}	F_D
Loop (4)		
Level controller →	M_B	F_B