

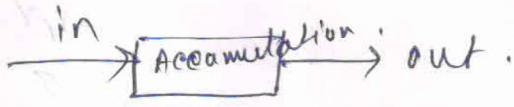
CAPE

(4)

In - out + generation = Accumulation of mass, momentum, Energy.

$H = U + PV$, ~~U~~ $U = \text{internal energy} = f(T)$.

Process Engg. design, operation, control, optimization, intensification



Steady state Accumulation = 0

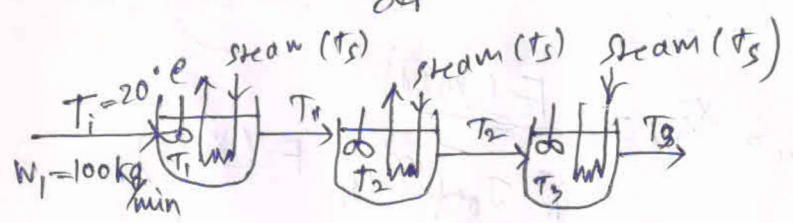
For ~~no~~ generation = 0. $in = out$.

1st

$$\frac{d(MC_p T_1)}{dt} = W C_p T_i - W C_p T_1 + UA (T_{\text{steam}} - T_1)$$

$$\text{2nd } \frac{d(MC_p T_2)}{dt} = W C_p (T_1 - T_2) + UA (T_{\text{steam}} - T_2)$$

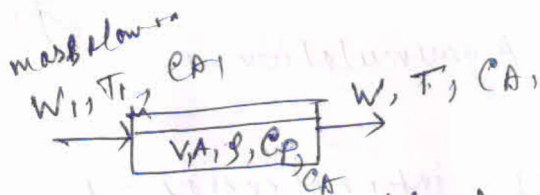
$$\text{3rd } \frac{d(MC_p T_3)}{dt} = W C_p (T_2 - T_3) + UA (T_{\text{steam}} - T_3)$$



A = Area of heat transfer for coil.

U = overall heat transfer coefficient

Himmelblau, D.M., Basic Principles & calculation in Chemical Engg. Prentice, Hall.
 Brack, A. Finlayson B.A., Introduction to Chemical Engineering computing.



$$\frac{d(V_A S)}{dt} = W_1 - W + \text{generation}$$

$$\frac{d(V_A S C_P T)}{dt} = W_1 C_P T_1 - W C_P T + \text{generation} \begin{matrix} \uparrow \\ -r_A V \end{matrix}$$

For heating system generation = $U A (T_s - T)$.

For reaction " " = Heat of reaction = $\Delta H_R (C_{AV}) (-r_A V)$.

$$-r_A = \frac{dC_A}{dt} = -k C_A$$

Species balance

$$\frac{d(C_A V M_w)}{dt} = \frac{W_1}{S} C_{A1} - \frac{W}{S} C_A - r_A V$$

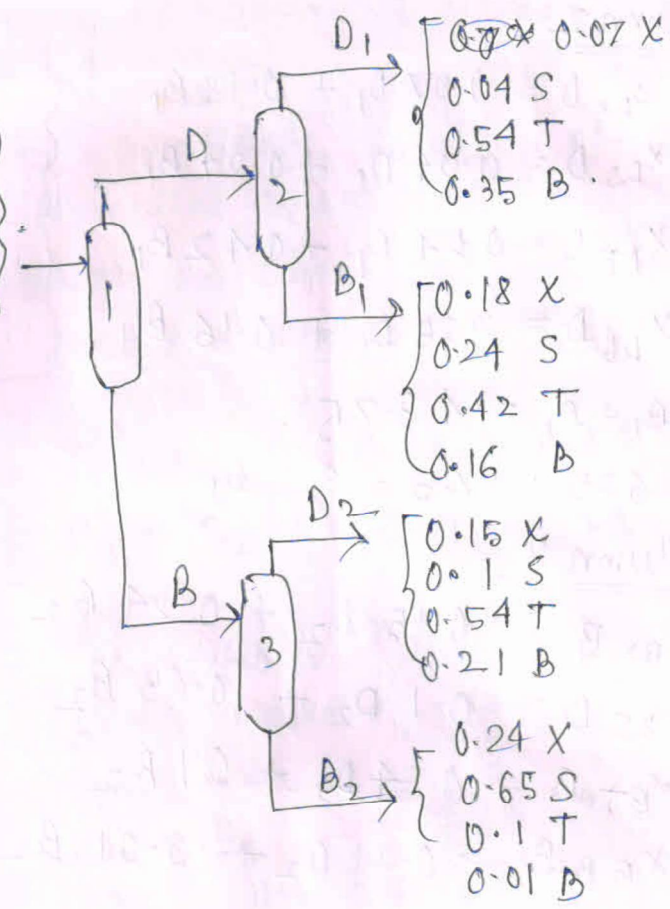
ChP2K13@gmail.com

$$\tilde{x}_{new} = \tilde{x}_{old} - \frac{1}{J} \left(\tilde{x}_{old} \right) F(\tilde{x}_{old})$$

$$J = \begin{bmatrix} \frac{\partial F_0}{\partial x_0} & \frac{\partial F_0}{\partial x_1} & \dots & \frac{\partial F_0}{\partial x_{n-1}} \\ \frac{\partial F_1}{\partial x_0} & \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_{n-1}} \end{bmatrix}$$

0.15 X
 0.25 S
 0.4 T
 0.2 B
 F = 70 mol/min

X = Xylene
 S = Styrene
 T = Toluene
 B = Benzene



Overall component balance

X: $0.07D_1 + 0.18B_1 + 0.15D_2 + 0.24B_2 = 0.15 \times 70$ — (A1)

S: $0.04D_1 + 0.24B_1 + 0.1D_2 + 0.65B_2 = 0.25 \times 70$ — (A2)

T: $0.54D_1 + 0.42B_1 + 0.54D_2 + 0.1B_2 = 0.4 \times 70$ — (A3)

B: $0.35D_1 + 0.16B_1 + 0.21D_2 + 0.01B_2 = 0.2 \times 70$ — (A4)

Overall material balance

$D_1 + B_1 + D_2 + B_2 = 70$ — (A5)

$D_1 = 26.25; B_1 = 17.50; D_2 = 8.7499$
 $B_2 = 17.4999;$

on column 2

(3)

$$\begin{aligned} X \rightarrow x_{Dx} D &= 0.07 D_1 + 0.18 B_1 \\ x_{Ds} D &= 0.04 D_1 + 0.24 B_1 \\ x_{Dt} D &= 0.54 D_1 + 0.42 B_1 \\ x_{Db} D &= 0.35 D_1 + 0.16 B_1 \end{aligned}$$

$$\begin{aligned} D &= B_1 + D_1 = 43.75 \\ &= 26.25 + 17.5 = B_1 + D_1 \end{aligned}$$

on column 3

$$\begin{aligned} X \quad x_{Bx} B &= 0.15 D_2 + 0.24 B_2 \\ S \quad x_{Bs} B &= 0.1 D_2 + 0.65 B_2 \\ T \quad x_{Bt} B &= 0.54 D_2 + 0.1 B_2 \\ B \quad x_{Bb} B &= 0.21 D_2 + 0.01 B_2 \end{aligned}$$

$$B = B_2 + D_2 = 8.7499 + 17.5 = \underline{\underline{26.25}}$$

APE: Review of numerical methods - convergence techniques, solution of linear and non-linear algebraic equations, solution of coupled ordinary differential equations. (1)

1. Introduction to chemical Engineering computing, Bruce A Finlayson, John Wiley & Sons, Inc.

$$F_1 = P_1 + F_2 \quad x^{(1)} \quad x^{(2)}$$

$$x^{(2)} F_2 = P_{2D} + P_{2B}$$

$$0.4 F_1 = P_1 \times 0.01 + F_2 X_{F_2}^T \quad x^{(5)}$$

$$0.4 F_1 = P_1 \times 0.99 + F_2 X_{F_2}^B \quad x^{(6)}$$

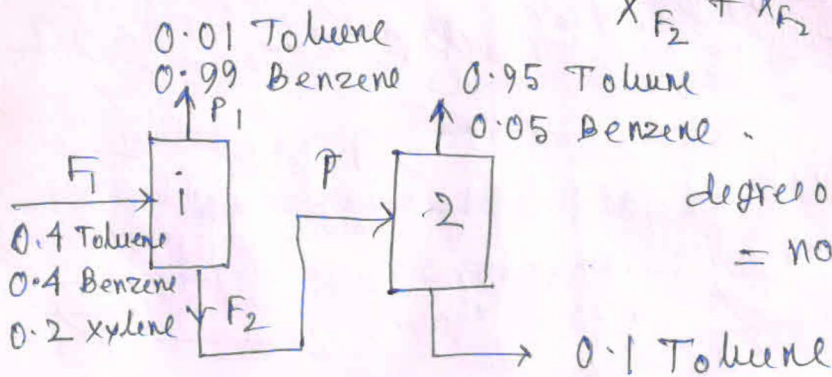
variables: x_1, x_2, x_3, x_4
 P_1, F_2, P_{2D}, P_{2B}
 x_5, x_6
 $X_{F_2}^T, X_{F_2}^B, X_{F_2}^X$

no of equations

$$F_2 X_{F_2}^T = 0.95 P_{2D} + 0.1 P_{2B}$$

$$F_2 X_{F_2}^B = 0.05 P_{2D} + 0.9 P_{2B}$$

$$X_{F_2}^T + X_{F_2}^B + X_{F_2}^X = 1$$



degree of freedom = no. of variable - no. of equations.

Initial condition

i.e. $\left\{ \begin{array}{l} F_2 = 500; P_{2D} = 250; P_{2B} = 250 \\ P_1 = 500; X_{F_2}^T = 0; X_{F_2}^B = 0.01 \end{array} \right.$

Sol $P_1 = 384.16 \text{ mol/min}$

$F_2 = 615.83 \text{ "}$

$P_{2D} = 393.61 \text{ "}$

$P_{2B} = 222.22 \text{ mol/min}$

$X_{F_2}^T = 0.6432, X_{F_2}^B = 0.03195, X_{F_2}^X = 0.3247$

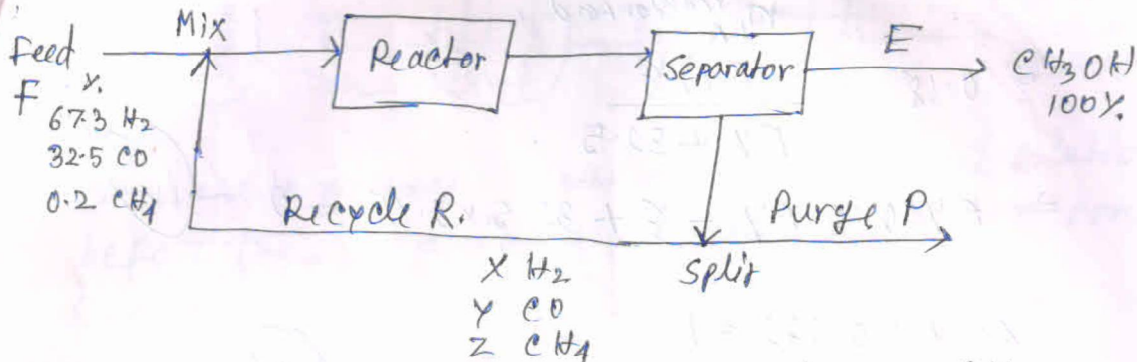
With fsolve after 8 iteration solutions are reached $\text{Prval} < 10^{-8}$

Prob Purge-Recycle-Reacting System

Steady State
Open Process



The once through conversion of CO in the reactor is 18%.



The purge stream is used to maintain the CH_4 concentration in the exit stream from the separator CH_4 (R & P) at no more than 3.2 mol%.

\therefore Assume $Z = 0.032$.

Select a basis $F = 100 \text{ mol/min}$.

The overall CH_4 balance

$$0.002 \times 100 = 0.032 P$$

$$\Rightarrow P = 6.25 \text{ mol/min}$$

Unknown variables: E, R, X, Y, ξ = extent of reaction.

Component balance for species i is

$$n_i^{\text{out}} = n_i^{\text{in}} + \nu_i \xi$$

$$\begin{aligned} \text{CO:} \quad 6.25 Y &= 0.325 \times 100 + (-1) \xi \\ 6.25 Y + \xi - 32.5 &= 0 \quad \text{--- (i)} \end{aligned}$$

$$\begin{aligned} \text{H}_2: \quad 6.25 X &= 0.673 \times 100 + (-2) \xi \\ 6.25 X + 2\xi - 67.3 &= 0 \quad \text{--- (ii)} \end{aligned}$$

$$\begin{aligned} \text{CH}_3\text{OH:} \quad E \times 1 &= 0 + 1 \times \xi \\ E - \xi &= 0 \quad \text{--- (iii)} \end{aligned}$$

Fractional conversion $\theta = 0.18 = 1$

$$\therefore 0.18 = \frac{-V_{LR} \xi}{n_{LR} \text{ reactor prod.}}$$

LR is CO here

$$\Rightarrow 0.18 = \frac{-(-1)\xi}{RY + 32.5}$$

$$\Rightarrow 0.18 RY - \xi + 32.5 \times 0.18 = 0 \quad \text{--- (iv)}$$

$$X + Y + 0.032 = 1$$

$$\Rightarrow X + Y + 0.032 - 1 = 0; \quad \text{--- (v)}$$

E, R, X, Y, ξ , 5 unknowns

initial condition

$x_1 \ x_2 \ x_3 \ x_4 \ x_5$
50, 50, 0.9, 0.1, 100.

With ξ solve after 19th iteration

solution is $E = 31.25$ mol/min (x_1)

$R = 705.55$ mol/min (x_2)

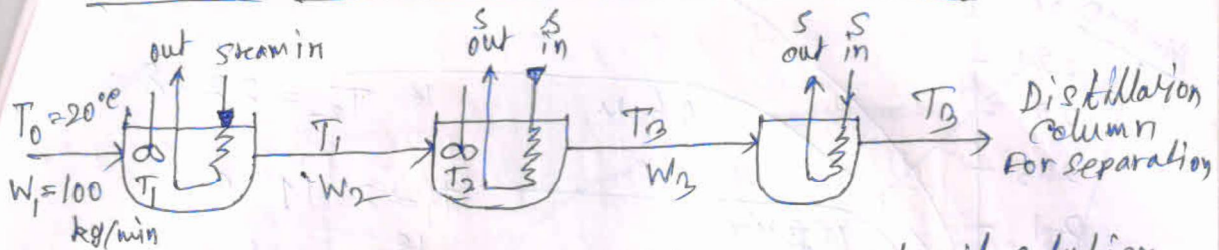
$X = 0.768$ (x_3)

$Y = 0.2$ (x_4)

$\xi = 31.25$ (x_5)

$\text{Error} < 10^{-10}$

Series of stirred tank heaters for oil heating



It is used to pre-heat a multicomponent oil solution before it is fed to a distillation column for separation.

$$c_p^{\text{oil}} = 2.0 \text{ kJ/kg} \cdot \text{K}, \quad W_1 = W_2 = W_3$$

$$Q = UA (T_{\text{steam}} - T) \quad \text{at steady state}$$

$$UA = 10 \text{ kJ/min/}^\circ\text{C}$$

$$W_{\text{in}} = W_{\text{out}}$$

$$T_s = 250^\circ\text{C}$$

$$M = \text{mass of oil in each tank} = 1000 \text{ kg}$$

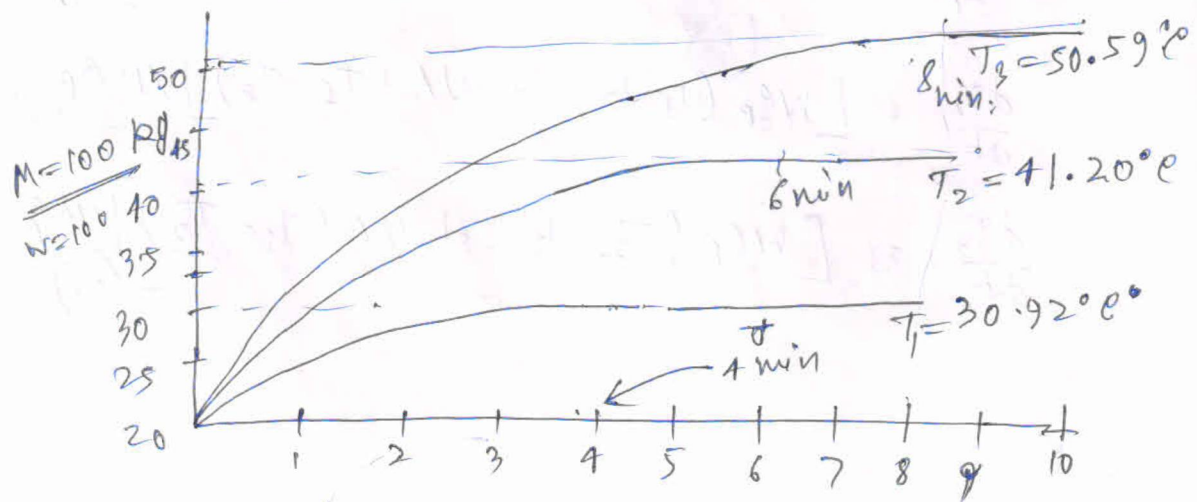
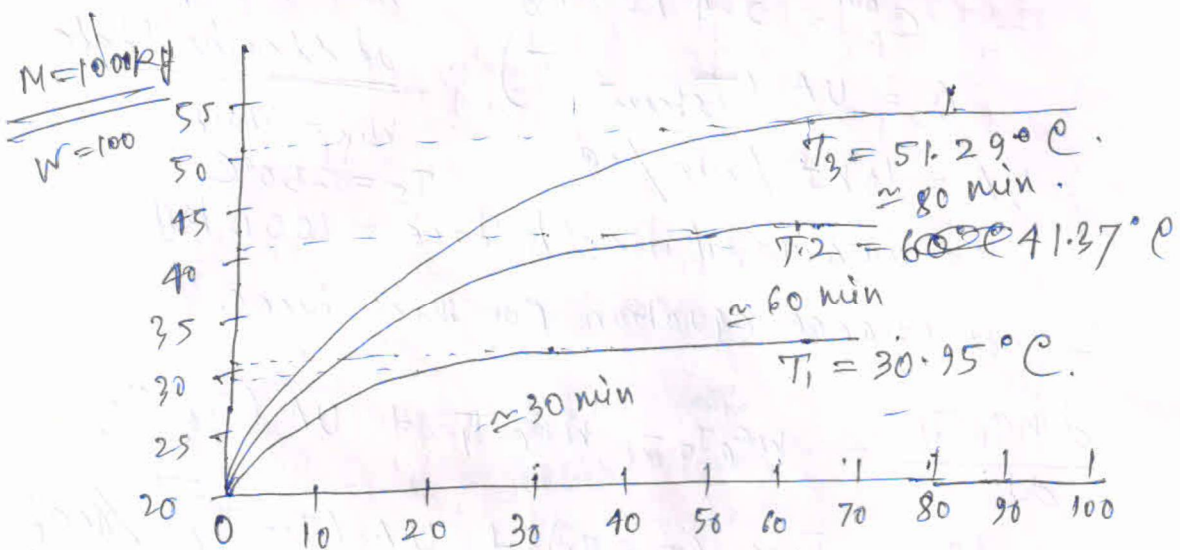
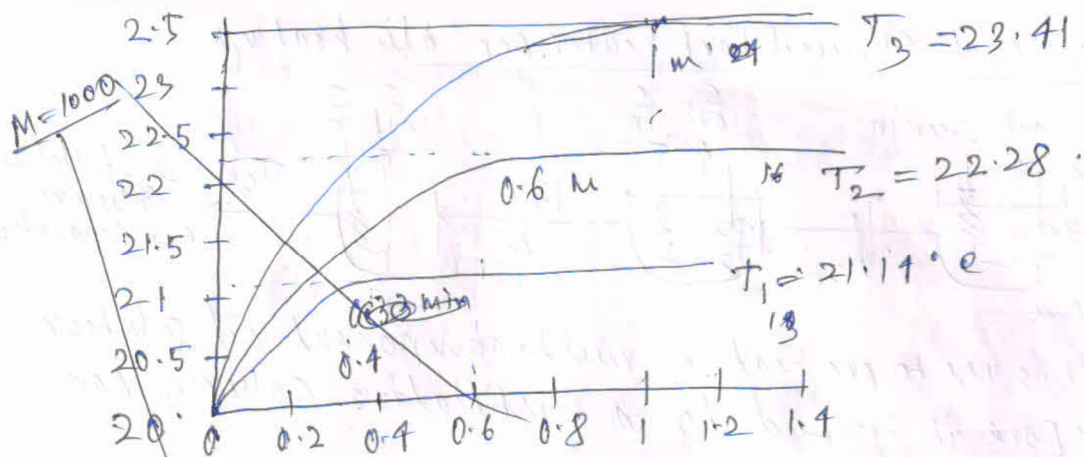
Energy balance equation for three tanks.

$$\frac{d(Mc_p T_1)}{dt} = Wc_p T_0 - Wc_p T_1 + UA (T_s - T_1)$$

$$\Rightarrow \frac{dT_1}{dt} = \frac{[Wc_p (T_0 - T_1) + UA (T_s - T_1)]}{M c_p} \quad \text{--- (i)}$$

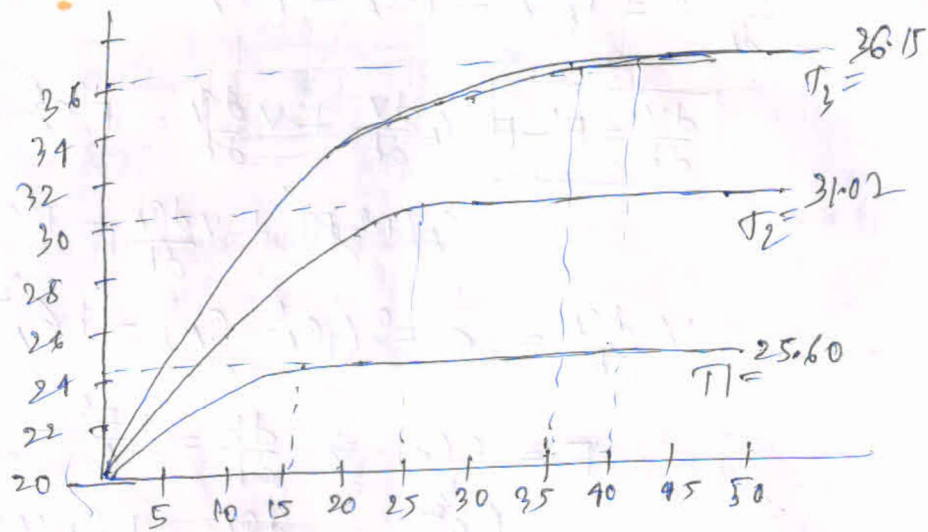
$$\frac{dT_2}{dt} = \frac{[Wc_p (T_1 - T_2) + UA (T_s - T_2)]}{M c_p} \quad \text{--- (ii)}$$

$$\frac{dT_3}{dt} = \frac{[Wc_p (T_2 - T_3) + UA (T_s - T_3)]}{M c_p} \quad \text{--- (iii)}$$



$$W=200$$

$$M=1000$$



ode 45 (@functionname, [0, tspan], [T1, T2, T3])

Runge-kutta 4th order for coupled ODE

$$\frac{dY}{dt} = F_1(Y, W, t); \quad \frac{dW}{dt} = F_2(Y, W, t)$$

with IC Y_0, W_0, t_0 .

After Δt time $t_1 = t_0 + \Delta t$.

$$Y_1 = Y_0 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

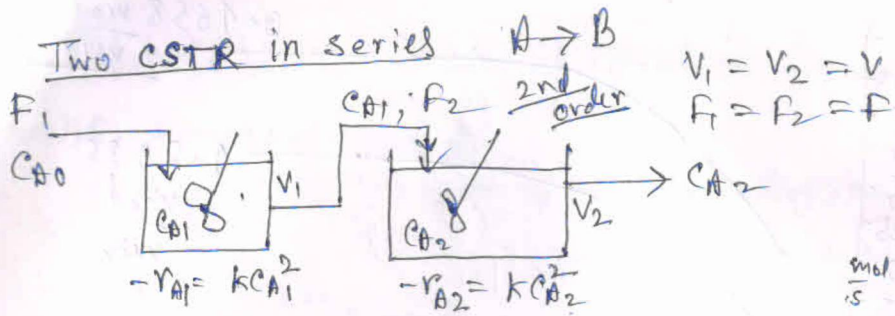
$$W_1 = W_0 + \frac{1}{6} (l_1 + 2l_2 + 2l_3 + l_4)$$

$$k_1 = F_1(Y_0, W_0, t_0) \Delta t, \quad l_1 = F_2(Y_0, W_0, t_0) \Delta t$$

$$k_2 = F_1\left(Y_0 + \frac{k_1}{2}, W_0 + \frac{l_1}{2}, t_0 + \frac{\Delta t}{2}\right) \Delta t, \quad l_2 = F_2\left(Y_0 + \frac{k_1}{2}, W_0 + \frac{l_1}{2}, t_0 + \frac{\Delta t}{2}\right) \Delta t$$

$$k_3 = F_1\left(Y_0 + k_2, W_0 + l_2, t_0 + \frac{\Delta t}{2}\right) \Delta t, \quad l_3 = F_2\left(Y_0 + k_2, W_0 + l_2, t_0 + \frac{\Delta t}{2}\right) \Delta t$$

$$k_4 = F_1(Y_0 + k_3, W_0 + l_3, t_0 + \Delta t) \Delta t, \quad l_4 = F_2(Y_0 + k_3, W_0 + l_3, t_0 + \Delta t) \Delta t$$



$k = \text{rate constant (2nd order)}$

$$\frac{\text{mol}}{\text{s}} = k \left(\frac{\text{mol}}{\text{lit}} \right)^2$$

$$k = \frac{\text{mol lit}^2}{\text{s mol}^2}$$

$$= \frac{\text{lit}^2}{\text{s mol}}$$

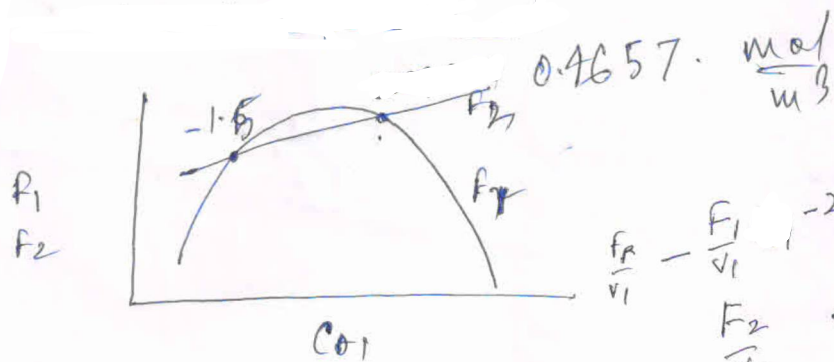
$$V_1 \frac{dC_{A1}}{dt} = F_1 (C_{A0} - C_{A1}) - kC_{A1}^2 V_1$$

$$\Rightarrow \frac{dC_{A1}}{dt} = \frac{F_1}{V_1} (C_{A0} - C_{A1}) - kC_{A1}^2 \quad \text{--- (i) = } F_1$$

$$\frac{dC_{A2}}{dt} = \frac{F_2}{V_2} (C_{A1} - C_{A2}) - kC_{A2}^2 \quad \text{--- (ii) = } F_2$$

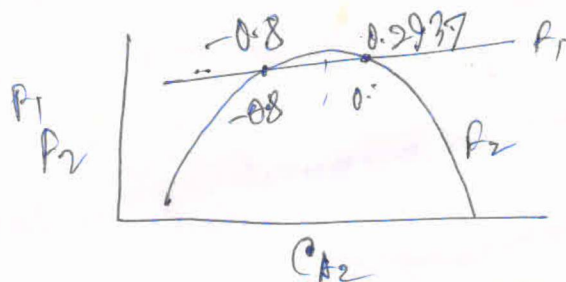
$F = 0.05, \quad V = 0.1, \quad k = 1; \quad C_{A0} = 0.9.$
 $\frac{\text{mol}}{\text{s}} \quad \text{m}^3 \quad \frac{\text{m}^6}{\text{s mol}^2} \quad \frac{\text{mol}}{\text{m}^3}$

$$\frac{F_1 C_{A1}}{V_1}$$



$$\frac{F_1}{V_1} - \frac{F_1}{V_1} - 2kC_{A1} = 0$$

$$\frac{F_2}{V_2} - \frac{F_2}{V_2} - 2kC_{A2}$$



$$\lambda = \frac{F_1}{V_1} - \frac{F_1}{V_1} - 2kC_{A1}$$

$$\frac{d(PV)}{dt} = \beta F^0 - \beta F$$

$$\frac{d(Vc_A)}{dt} = F^0 c_A^0 + F c_A - k c_A^2$$

$$\boxed{\frac{dV}{dt} = F^0 - F}; c_A \frac{dV}{dt} + V \frac{dc_A}{dt} = F^0 c_A^0 - F c_A - k c_A^2$$

$$c_A (F^0 - F) + V \frac{dc_A}{dt} = F^0 c_A^0 - F c_A - k c_A^2$$

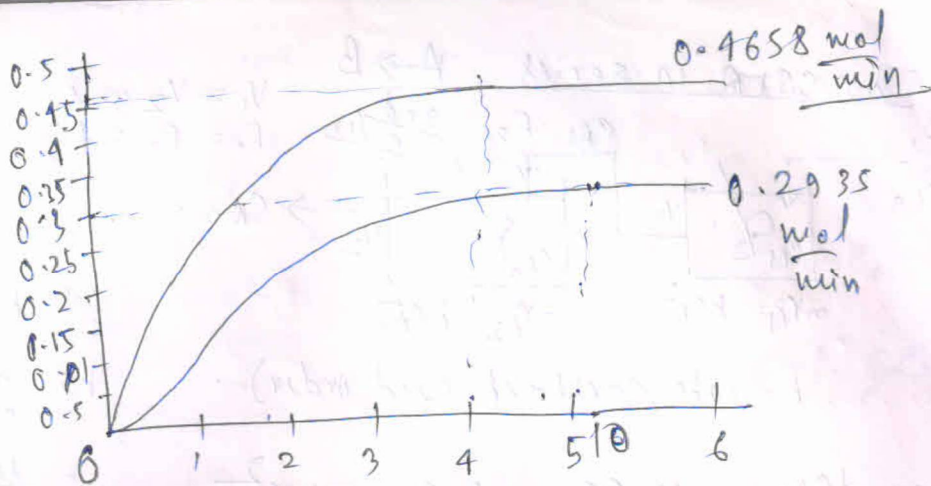
$$V \frac{dc_A}{dt} = F^0 (c_A^0 - c_A) - k c_A^2$$

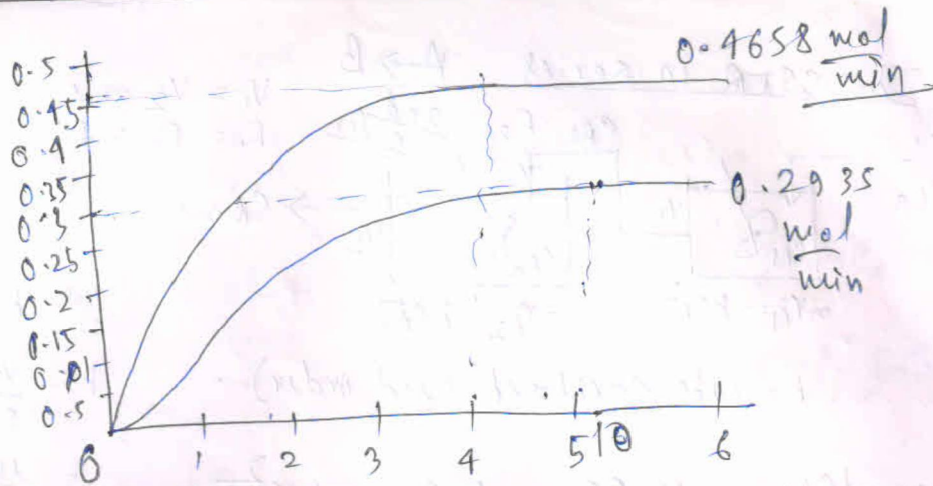
$$F = F(h) \Rightarrow \frac{dh}{dt} = \frac{F^0}{A} - \frac{R h^{3/2}}{A}$$

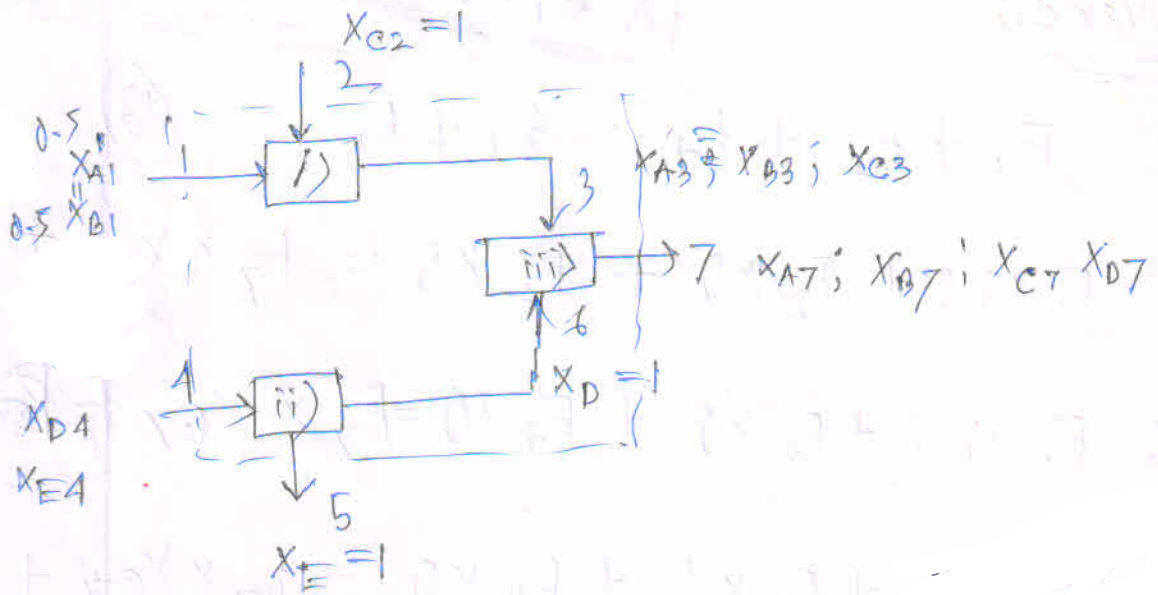
$$= R h^{1/2}$$

$$\underline{V = hA}$$

$$\frac{dc_A}{dt} = \frac{1}{hA} [F^0 (c_A^0 - c_A) - k c_A^2]$$







(i) $F_1 + F_2 = F_3$ — (1)

A: $F_1 \times 0.5 + F_2 \times 0 = F_3 \times X_{A3}$ — (2)

B: $F_1 \times 0.5 + F_2 \times 0 = F_3 \times X_{B3}$ — (3)

C: $F_1 \times 0 + F_2 \times 1 = F_3 \times X_{C3}$ — (4)

(ii) $F_4 = F_5 + F_6$ — (5)

D: $F_4 \times X_D = F_5 \times 0 + F_6 \times 1$ — (6)

E: $F_4 \times X_E = F_5 \times 1 + F_6 \times 0$ — (7)

(iii) $F_3 + F_6 = F_7$ — (8)

A: $F_3 \times X_{A3} + F_6 \times 0 = F_7 \times X_{A7}$ — (8)

B: $F_3 \times X_{B3} + F_6 \times 0 = F_7 \times X_{B7}$ — (9)

C: $F_3 \times X_{C3} + F_6 \times 0 = F_7 \times X_{C7}$ — (10)

D: $F_3 \times 0 + F_6 \times 1 = F_7 \times X_{D7}$ — (11)

~~$X_{A3} = X_{B3}$~~ — (2)

Overall

18 equations, 18 variables

$$F_1 + F_2 + F_4 = F_7 + F_5 \quad - \quad (13)$$

$$A: F_1 \times 0.5 + F_2 \times 0 + F_4 \times 0 = F_7 \times X_{A7} + F_5 \times 0 \quad - \quad (14)$$

$$B: F_1 \times 0.5 + F_2 \times 0 + F_4 \times 0 = F_7 \times X_{B7} + F_5 \times 0 \quad - \quad (15)$$

$$C: F_1 \times 0 + F_2 \times 1 + F_4 \times 0 = F_7 \times X_{C7} + F_5 \times 0 \quad - \quad (16)$$

$$D: F_1 \times 0 + F_2 \times 0 + F_4 \times X_{D4} = F_7 \times X_{D7} + F_5 \times 0 \quad - \quad (17)$$

$$E: F_1 \times 0 + F_2 \times 0 + F_4 \times X_{E4} = F_7 \times 0 + F_5 \times 1 \quad - \quad (18) - 5$$

(14)

(18) Equations

variables

$F_1, F_2, F_3, F_4, F_5, F_6, F_7$
→ (Flow rates)

16

$X_{A3}, X_{B3}, X_{C3}, X_{A7}, X_{B7}, X_{C7}, X_{D7}$

$X_{D4}, X_{E4}, X_{A1}, X_{B1} \rightarrow 11$

compositions.

$$X_{A1} + X_{B1} = 1 \quad - \quad (15)$$

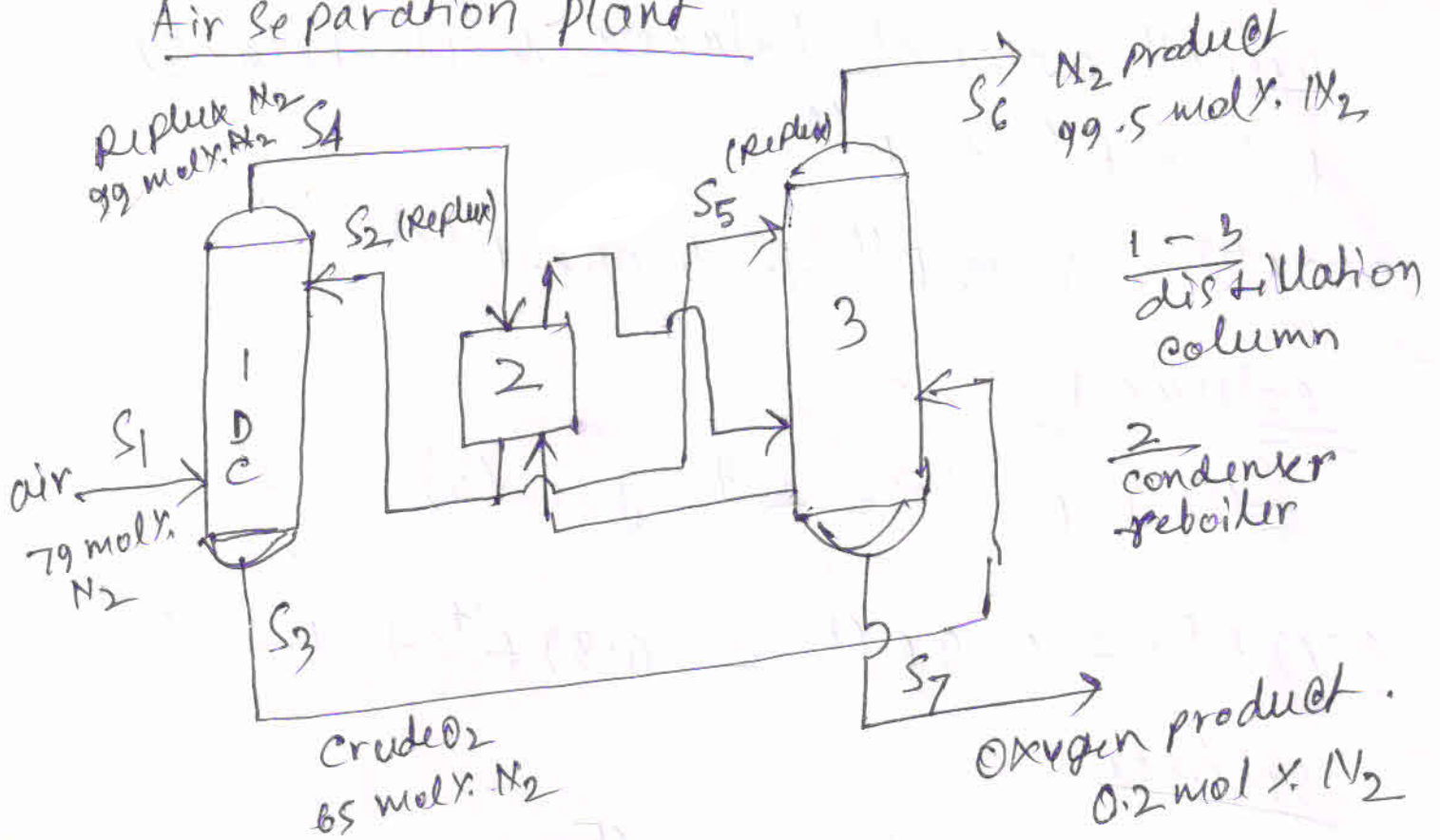
$$X_{A3} + X_{B3} + X_{C3} = 1 \quad -$$

→ 18 variables

$$(17) \quad X_{A7} + X_{B7} + X_{C7} + X_{D7} = 1 \quad - \quad (17)$$

$$X_{D4} + X_{E4} = 1 \quad - \quad (18)$$

Air separation plant



$$\begin{aligned}
 x_2^{(1)} &= 0.21 & 1 &\rightarrow N_2 \\
 x_2^{(3)} &= 0.35 & 2 &\rightarrow O_2 \\
 x_2^{(4)} &= 0.01 = x_2^{(2)} = x_2^{(5)} \\
 x_2^{(6)} &= 0.005, & x_2^{(7)} &= 0.998
 \end{aligned}$$

$F^{(5)} = 3 F^{(2)}$ given

$$F_7 = \underline{\underline{226.44 \text{ mol}}}$$

Reflux to the 2nd column is three times that of the first column.

Overall material balance & N_2 balance

$$F^{(1)} = F^{(6)} + F^{(7)}$$

$$0.79 F^{(1)} = 0.995 F^{(6)} + 0.002 F^{(7)}$$

column 1

$$F^{(1)} + F^{(2)} = F^{(4)} + F^{(3)}$$

$$0.79 F^{(1)} + 0.99 F^{(2)} = 0.99 F^{(4)} + 0.65 F^{(3)}$$

splitter

$$F^{(4)} = F^{(2)} + F^{(5)}$$

$$F^{(5)} = 0.3 F^{(2)}$$

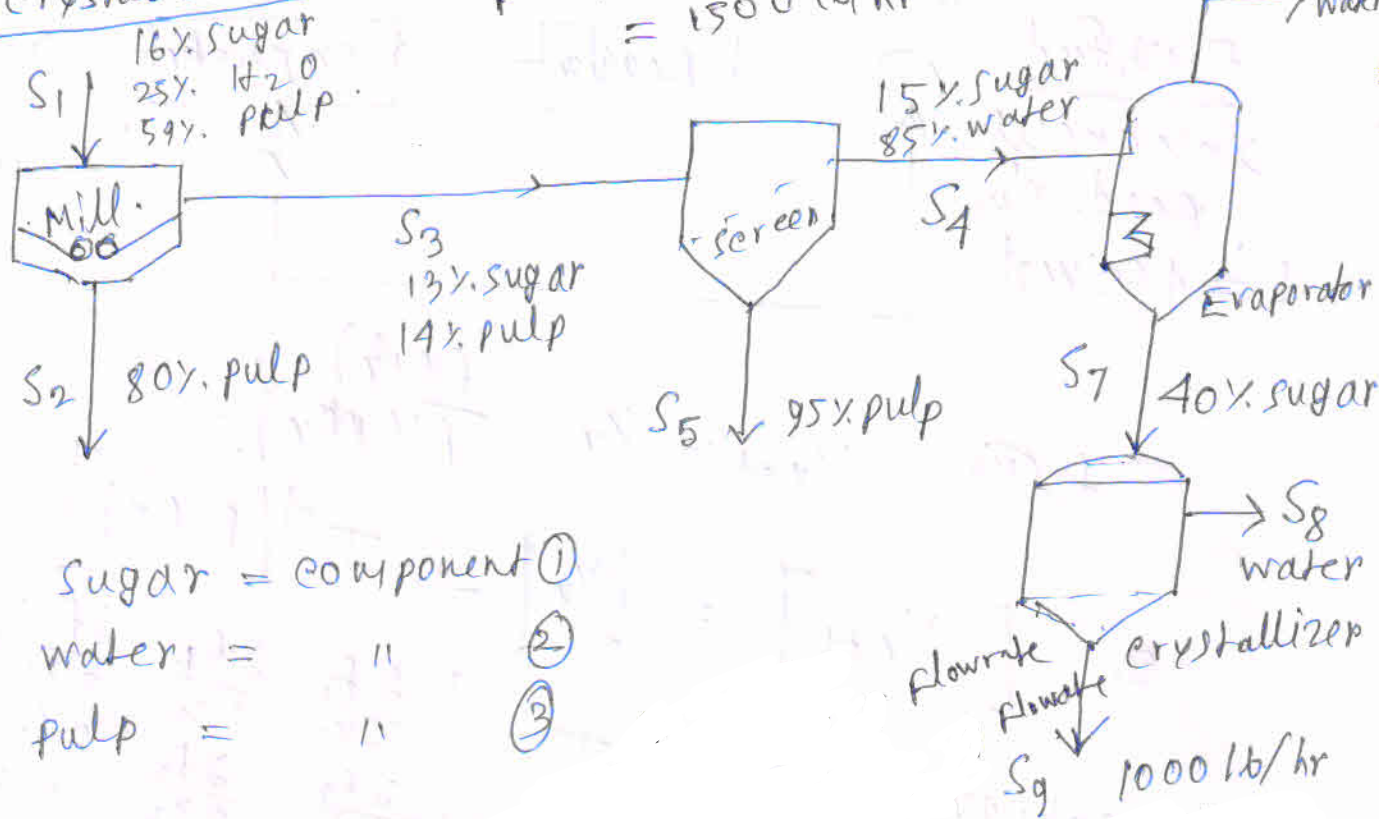
$$F^{(1)} \dots F^{(7)}$$

$$F^{(1)} = \frac{840}{183} \times 2500$$

$$F^{(7)} = 2500 \text{ lb/hr}$$

$$F^{(9)} = 2500 \times \frac{0.6}{1.0} = 1500 \text{ lb/hr}$$

Sugar crystallization



Sugar = component ①
 water = " ②
 pulp = " ③

$$\begin{cases} 0.16 F^{(1)} = w_1^{(2)} F^{(2)} + 0.13 F^{(3)} & \text{mill, sugar} \\ 0.25 F^{(1)} = w_2^{(2)} F^{(2)} + 0.73 F^{(3)} & \text{" , water} \\ 0.59 F^{(1)} = 0.8 F^{(2)} + 0.14 F^{(3)} & \text{" , pulp} \\ w_1^{(2)} + w_2^{(2)} + 0.8 = 1 & \text{Stream 2} \end{cases}$$

$$\begin{cases} 0.13 F^{(3)} = 0.15 F^{(4)} + w_1^{(5)} F^{(5)} & \text{Screen, sugar} \\ 0.73 F^{(3)} = 0.85 F^{(4)} + w_2^{(5)} F^{(5)} & \text{Screen, water} \\ 0.14 F^{(3)} = 0.95 F^{(5)} & \text{Screen, pulp} \\ w_1^{(5)} + w_2^{(5)} + 0.95 = 1 & \text{Stream 5} \end{cases}$$

$$\begin{cases} 0.85 F^{(4)} = F^{(6)} + 0.6 F^{(7)} & \text{Evaporator, water} \\ 0.15 F^{(4)} = 0.4 F^{(7)} & \text{" , sugar} \\ 0.6 F^{(7)} = F^{(8)} & \text{Crystallizer, water} \\ 0.4 F^{(7)} = 1000 & \text{" , sugar} \end{cases}$$

$$F^{(7)} = \frac{1000}{0.4} = 2500$$

Single reaction - ideal solution

outlet $\rightarrow N_i' = N_i(F) + \frac{v_i'}{-v_A} x_A N_A(F)$ $\left[\begin{matrix} \rho_i \\ \rho_i \end{matrix} = \frac{x_A N_A}{-v_A} \right]$

A is limiting reactant.

$x_A =$ conversion.

$Q + \sum N_i(F) H_i(T, P)$
all species at inlet.

$= \sum \left[N_i(F) + \frac{v_i' x_A N_A(F)}{-v_A} \right] H_i(T, P)$
all species at outlet.

$Q + \frac{x_A N_A(F)}{-v_A} \left[- \sum v_i H_i(T, P) \right]$

$= \sum N_i(F) \left[H_i(T, P) - H_i(T, P^A) \right]$

$N_{out}^i = N_{in}^i + \rho_i$

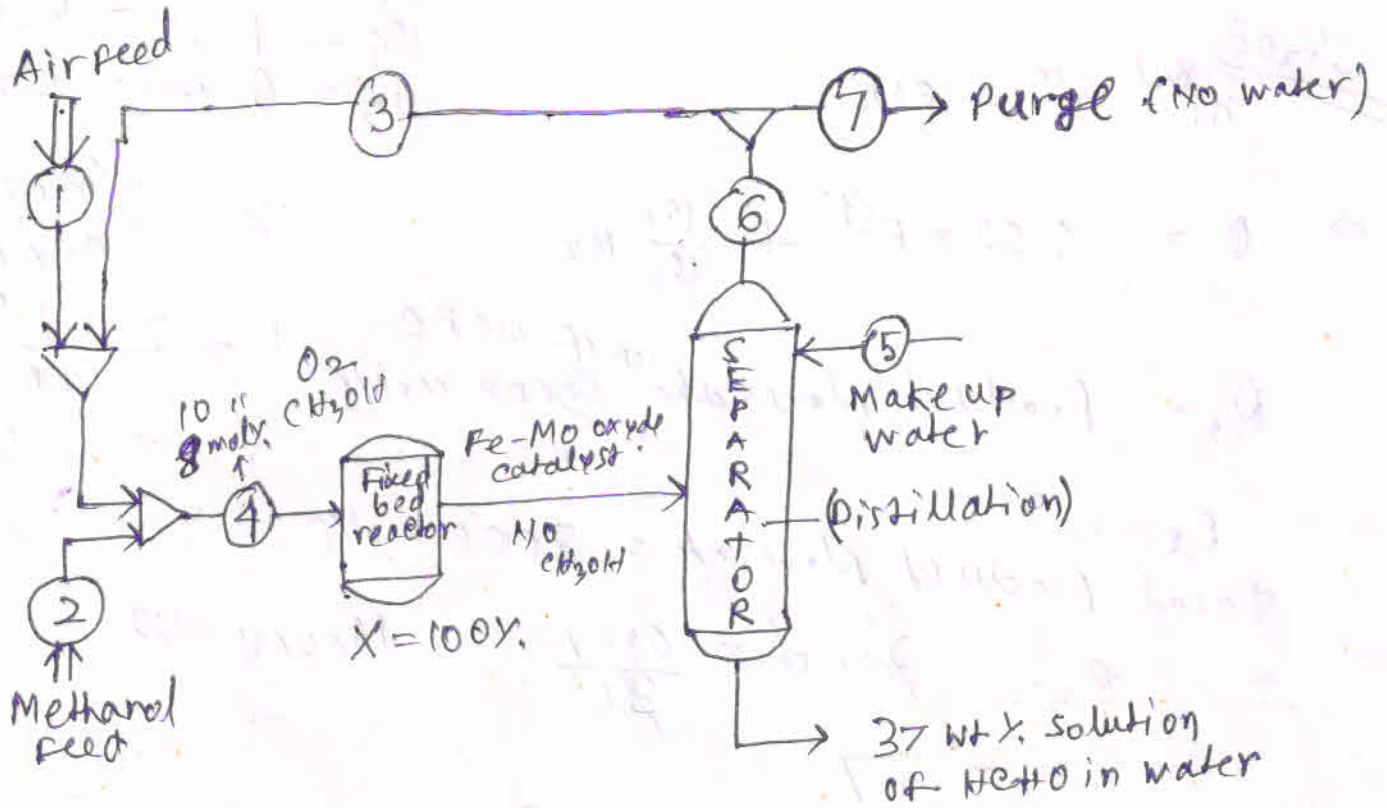
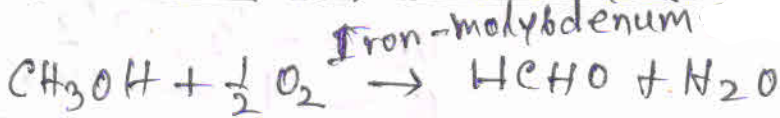
$\rho_A = \frac{-v_A \rho}{F_{LR in}}$

$\rho = \frac{x_A F_{LR}}{-v_A}$

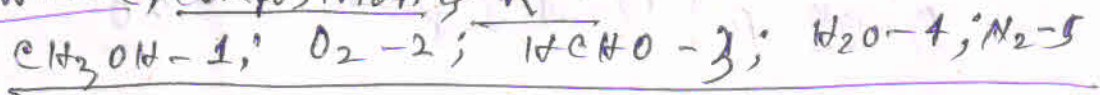
$N_{out}^i = N_{in}^i + \frac{v_i'}{-v_A} x_A F_{LR}$

Production of formaldehyde

oxyde catalyst.



calculate flowrate, composition of R.



$$X_2^{(1)} = 0.21; X_5^{(1)} = 0.79; X_1^{(4)} = 0.08; X_2^{(4)} = 0.1; X_5^{(4)} = 0.82$$

Mixers before reactor

CH₃OH: $F^{(2)} = X_1^{(4)} F^{(4)}$
 $F^{(2)} = 0.08 F^{(4)}$ — (1)

O₂: $X_2^{(1)} F^{(1)} + X_2^{(3)} F^{(3)} = X_2^{(4)} F^{(4)}$
 $0.21 F^{(1)} + X_2^{(3)} F^{(3)} = 0.1 F^{(4)}$ — (2)

N₂: $X_5^{(1)} F^{(1)} + X_5^{(3)} F^{(3)} = X_5^{(4)} F^{(4)}$
 $0.79 F^{(1)} + X_5^{(3)} F^{(3)} = 0.82 F^{(4)}$ — (3)

definition $X_5^{(3)} + X_2^{(3)} = 1$ — (4)

Reactor + separator.

+

C_{H_2O}
 $n_i^{out} = n_i^{in} + \xi v_i$

- $v_1 = -1 \rightarrow C_{H_2O}$
- $v_2 = -1/2 \rightarrow O_2$
- $v_3 = 1 \rightarrow H_2O$
- $v_4 = 1 \rightarrow H_2O$
- $v_5 = 0 \rightarrow N_2$

$\Rightarrow 0 = 0.08 \times F^{(4)} + \frac{v_1}{v_3} R_3$

$X = \frac{-(v_{LR}) \xi}{n_{LR} \text{ feed input}}$

$R_3 =$ Product flow rate of H_2O mol/s.

$1 = \frac{-(-1) \xi}{n_{LR} \cdot n_{LR} \xi}$

$\xi = n_{LR} \cdot 0.9$

Total product flow rate = 3000 mol/s.

$\therefore R_3 = 3000 \times \frac{0.37}{30}$ $M_{H_2O} = 30$
 $= 37$

$0.08 F^{(4)} - F^{(8)} \frac{0.37}{30} = 0$ — (5)

H_2O :

$\frac{0.63 \times F_8}{18} = F^{(5)} + \frac{v_4}{v_3} F^{(8)} \times \frac{0.37}{30}$

$\frac{0.63}{18} F_8 = F^{(5)} + F^{(8)} \frac{0.37}{30}$ — (6)

O_2 :

$X_2^{(6)} F^{(6)} = 0.1 F^{(4)} + \frac{v_2}{v_3} F^{(8)} \frac{0.37}{30}$

$X_2^{(6)} F^{(6)} = 0.1 F^{(4)} - \frac{1}{2} F^{(8)} \frac{0.37}{30}$ — (7)

N_2

$X_5^{(6)} F^{(6)} = 0.82 F^{(4)}$ — (8)

Splitter

$F^{(3)} + F^{(7)} = F^{(6)}$ — (9)

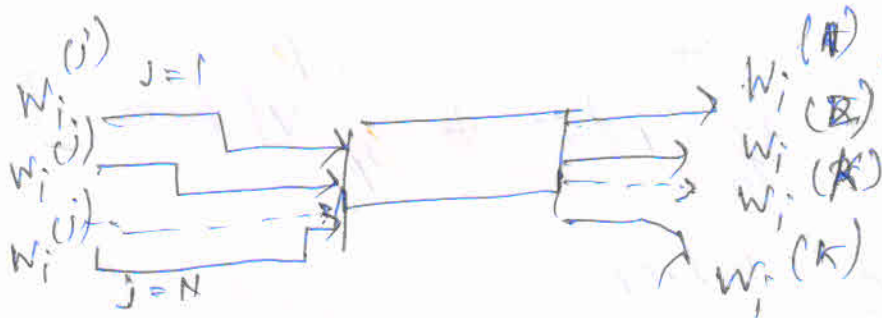
$F_0^1, F_0^2, F_0^3, F_0^4, F_0^5, F_0^6, F_0^7, F_0^8$ 8 variables $x_2^3, x_5^3, x_2^6, x_5^6$

in macroscopic view

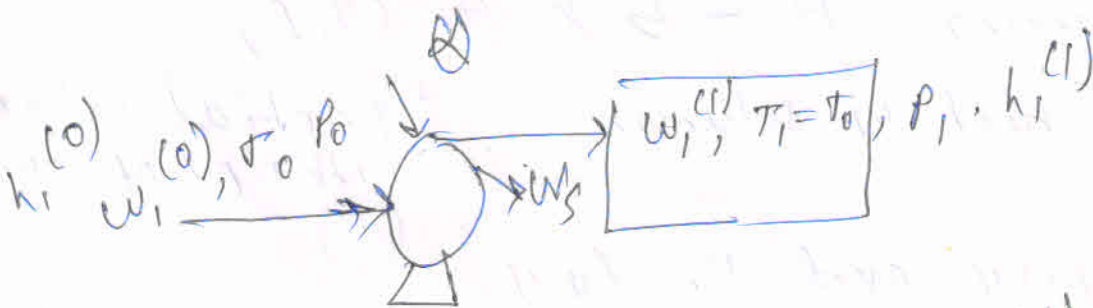


$$Q + \sum W_i^{(i)} \left[\bar{h}_i + \frac{1}{2} v^2 + g h \right]_{\text{stream } j}$$

$$= \sum W_i^{(k)} \left[\bar{h}_i + \frac{1}{2} v^2 + g h \right]_{\text{stream } k}$$



+ W_S
shaft work
 $W_S = \text{shaft work}$



$$Q + \sum W_i^{(0)} \bar{h}_i^{(0)} = \sum W_i^{(1)} \bar{h}_i^{(1)} + W_S$$

Enthalpy of liquid & solid

$$H = H(T_0, P_0) + \int_{T_0}^T \left(\frac{\partial H}{\partial T} \right)_P dT + \int_{P_0}^P \left(\frac{\partial H}{\partial P} \right)_T dP$$

$$H = \int_{T_0}^T c_p dT + \int_{P_0}^P \left[v + \left(\frac{\partial v}{\partial T} \right)_P \right] dP + H(T_0, P_0)$$

$$\alpha_T = \frac{1}{v_T} \left(\frac{\partial v}{\partial T} \right)_P = \text{coefficient of thermal expansion,}$$

$$H = H(T_0, P_0) + \int_{T_0}^T c_p dT + \int_{P_0}^P v [1 - \alpha_T T] dP$$

Non ideal liquid solution.

$$\Delta H_{\text{mixing}} = H - \sum x_i \tilde{H}_i^+(T, P)$$

↗ heat of solution

↘ partial molar enthalpy of component
(i)

Energy flow and 1st law

in flow systems

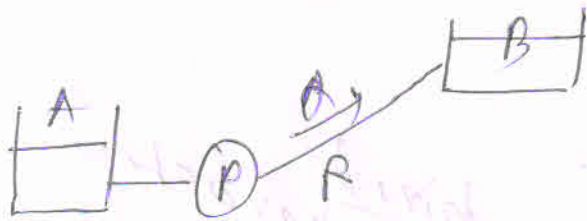
total energy = kinetic + potential + internal + work done on the body by pressure force

$$= \frac{1}{2} v^2 + gh + u + \frac{P}{\rho} \quad \text{Energy unit mass}$$

$$\frac{J}{kg} \quad \frac{kg}{m^3}$$

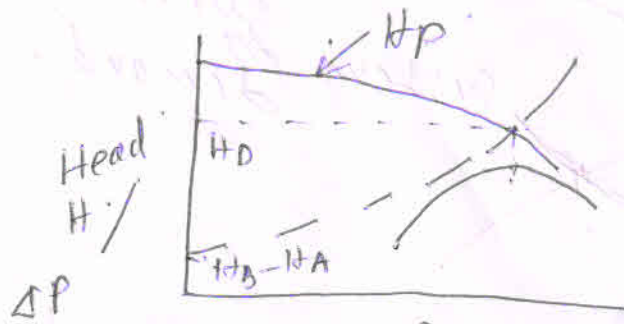
$$u + \frac{P}{\rho} = \left(u + \frac{Pv}{m} \right) = h; \\ = h; = \text{enthalpy/unit mass}$$

$$= \frac{1}{2} v^2 + gh + h;$$



$$W_p = \frac{\gamma Q H_p}{\eta}$$

$H_D =$ demand head.



$$f = \frac{8 \nu}{v^2}$$

assume kinetic energy parts are negligible

$$H_B - H_A = \sum h_f = \left(f \frac{L}{D} \frac{1}{2gA^2} + \frac{\sum K}{2gA^2} \right) Q^2$$

$$h_L = \frac{f L v^2}{D 2g}$$

Darcy-Weisbach eqn

$$\frac{P_B}{\rho g} + H_B - \frac{P_A}{\rho g} - Z_A + h_L = H_p$$

$$H_B - H_A + h_L = H_p$$

$$H_B - H_A +$$

$$R Q^2 = H_p$$

put $H_p =$ at maximum efficiency

colebrook-white

$$\frac{1}{\sqrt{f_D}} = -2 \log \left(\frac{2.5}{Re \sqrt{f_D}} + \frac{2.51}{Re \sqrt{f_D}} \right)$$

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon}{3.7 D \sqrt{f}} + \frac{2.51}{Re \sqrt{f}} \right)$$

Moody

$$f = 0.0055 \left[1 + \left(2 \times 10^4 \frac{\epsilon}{D} + \frac{10^6}{Re} \right)^{1/3} \right]$$

$$\frac{\epsilon}{D} = 0 - 0.01$$

Head loss h_L is
in a pipeline

$$h_L = f \frac{L}{D} \frac{v^2}{2g} \quad [\text{Darcy-Weisbach equation}]$$

valid both for laminar & turbulent flow.

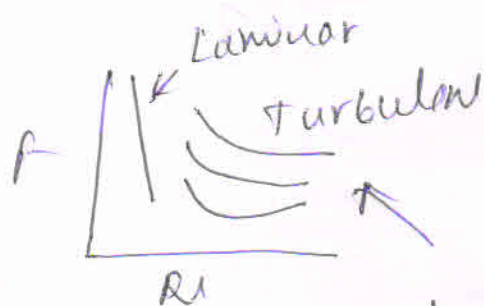
for laminar flow $f = \frac{64}{Re}$.

f = friction factor.

$$\frac{-W_s}{w \dot{Q}} = \frac{v_2^2}{2g} + \frac{P_2}{\rho g} + z_2 - \frac{P_1}{\rho g} - \frac{v_1^2}{2g} - z_1 + h_L$$

$$h_L = k \frac{v^2}{2g}, \quad k = \text{loss coeff.}$$

f = function $\left(\frac{e}{D}, \frac{v D \rho}{\mu} \right)$. $\frac{e}{D}$ relative roughness.



It is presented in Moody diagram.

$$h_L = 1.07 \frac{Q^2 L}{g D^5} \left[\ln \left[\frac{e}{3.7D} + 4.62 \left(\frac{v D}{\nu} \right)^{0.9} \right] \right]^2$$

$$10^{-6} < \frac{e}{D} < 10^{-2}, \quad \underline{3000 < Re < 3 \times 10^8}$$

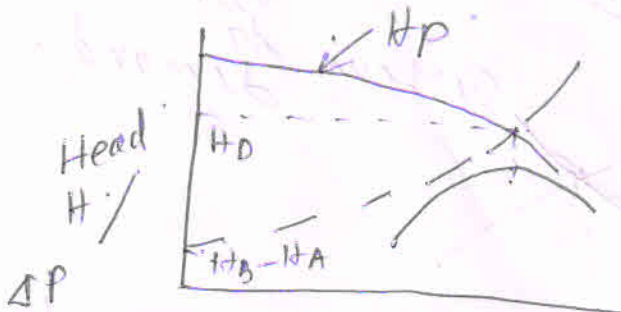
$$h_L = \frac{v^2}{2g} \left[\underbrace{f \frac{L}{D}}_{\text{straight pipe}} + \underbrace{\sum K}_{\substack{\text{bending, expansion,} \\ \text{contraction}}} \right]$$



$$W_p = \frac{\gamma Q H_p}{\eta}$$

$H_D = \text{demand head.}$

$$f = \frac{16}{Re} \nu^2$$



assume kinetic energy parts are negligible

$$H_B - H_A = \sum h_f = \left(f \frac{L}{D} \frac{1}{2gA^2} + \frac{\sum K}{2gA^2} \right) Q^2$$

$$h_L = \frac{f L V^2}{D 2g}$$

Darcy-Weisbach eqn

$$\frac{P_B}{\rho g} + H_B - \frac{P_A}{\rho g} - z_A + h_L = H_p$$

$$H_B - H_A + h_L = H_p$$

$$H_B - H_A + K Q^2 = H_p$$

put $H_p = \text{at maximum efficiency}$

colebrook-white

$$\frac{1}{\sqrt{f_D}} = -2 \log \left(\frac{2.51}{Re \sqrt{f_D}} + \frac{\epsilon}{3.7D} \right)$$

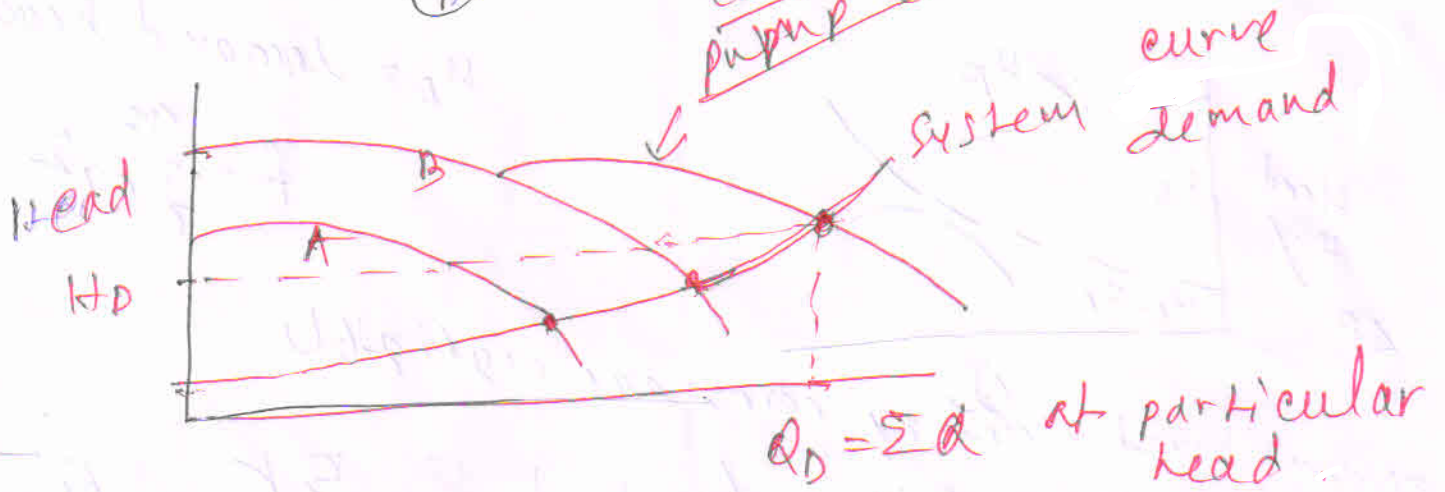
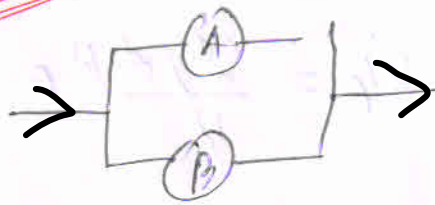
$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon}{3.7 D n} + \frac{2.51}{Re \sqrt{f}} \right)$$

Moody

$$f = 0.0055 \left[1 + \left(2 \times 10^{-4} \frac{\epsilon}{D} + \frac{10^6}{Re} \right)^{1/3} \right]$$

$$\frac{\epsilon}{D} = 0 - 0.01$$

parallel



Series

Pump combined

$H_D = \sum H$
at particular Q

