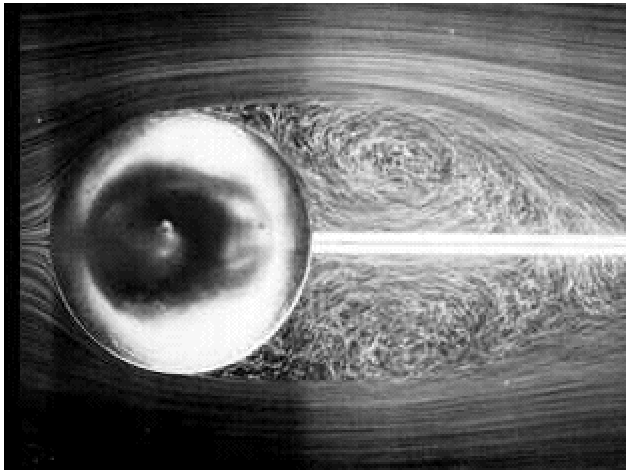
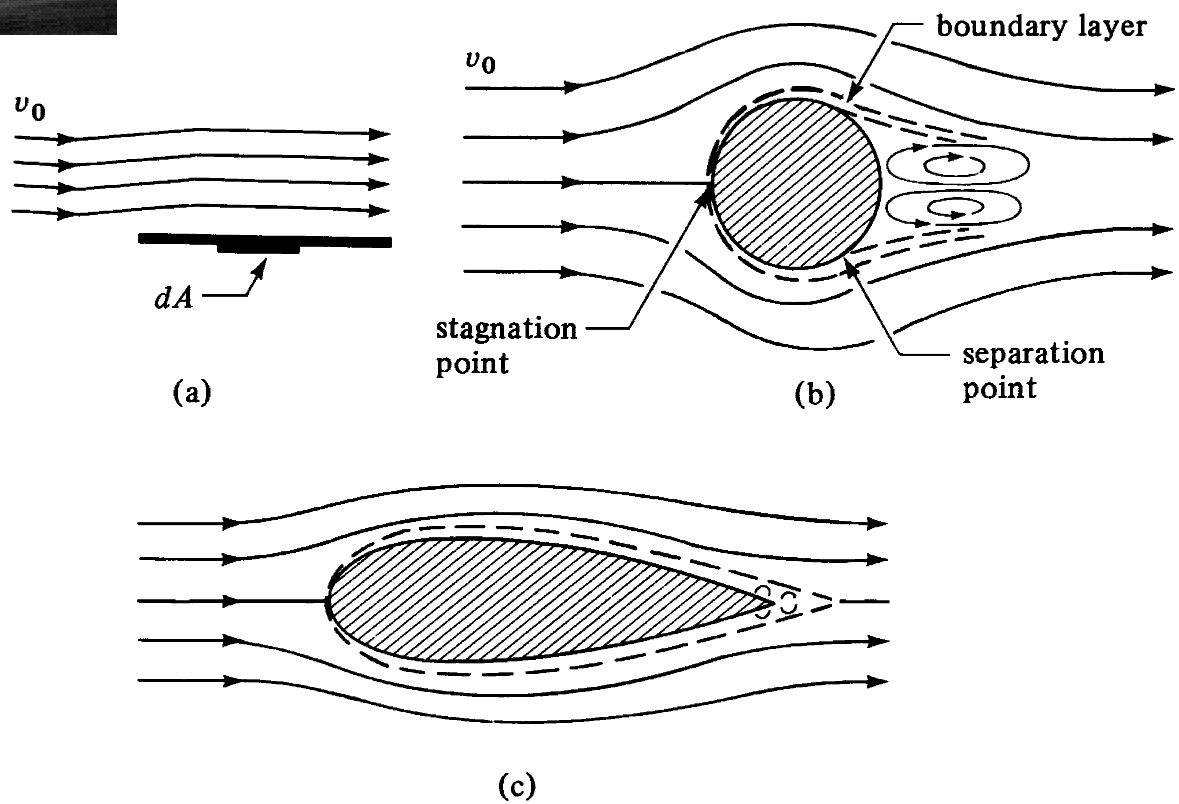
Module # 4

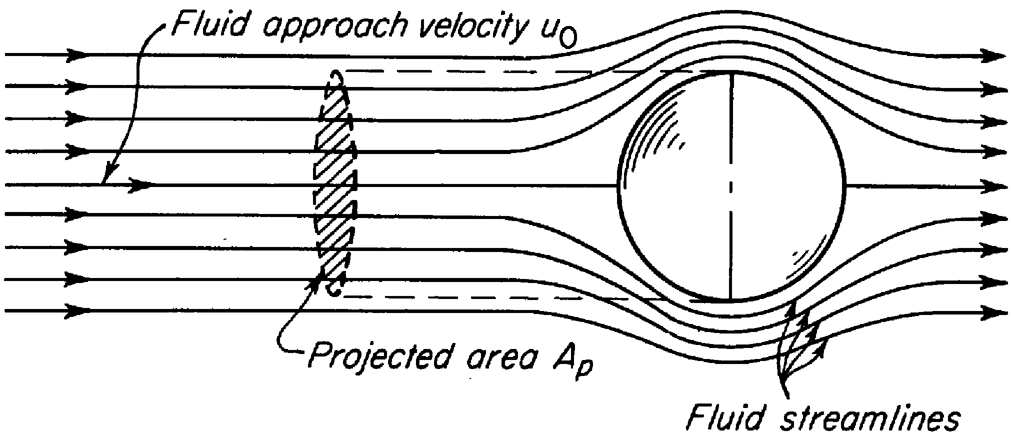
Flow Past of Immersed Bodies

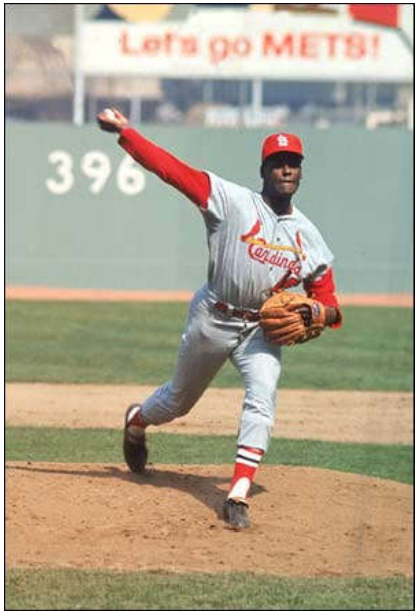
Incompressible Flow





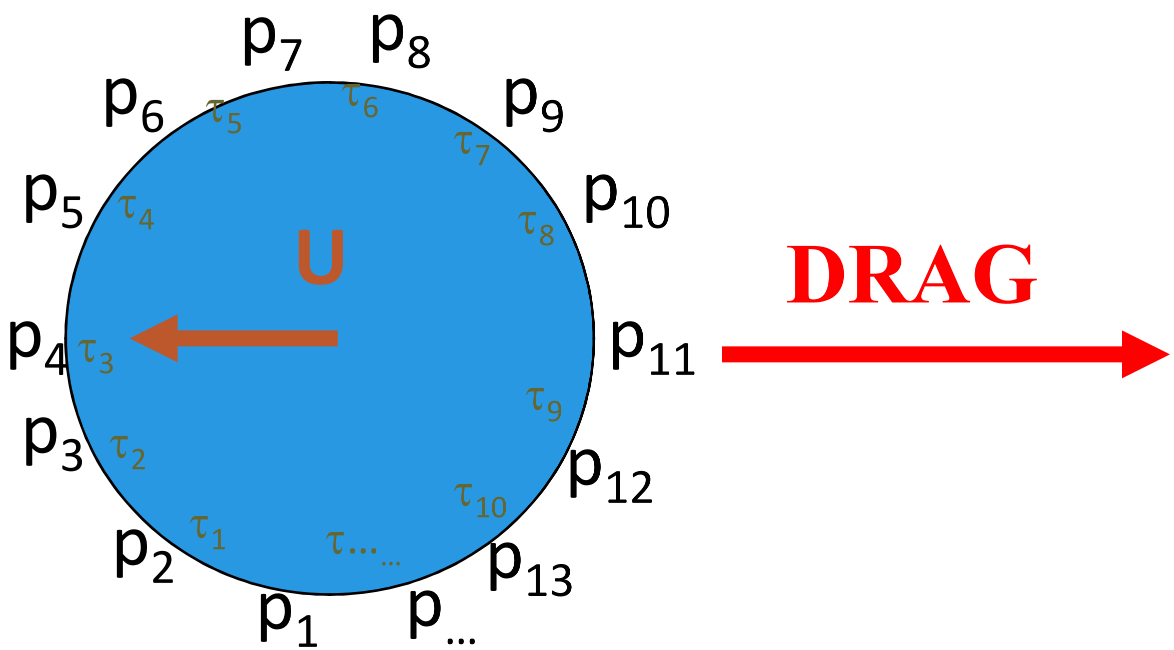
Flow Around Objects





FLUID FLOW ABOUT IMMERSED BODIES

Drag due to surface stresses composed of normal (pressure) and tangential (viscous) stresses.



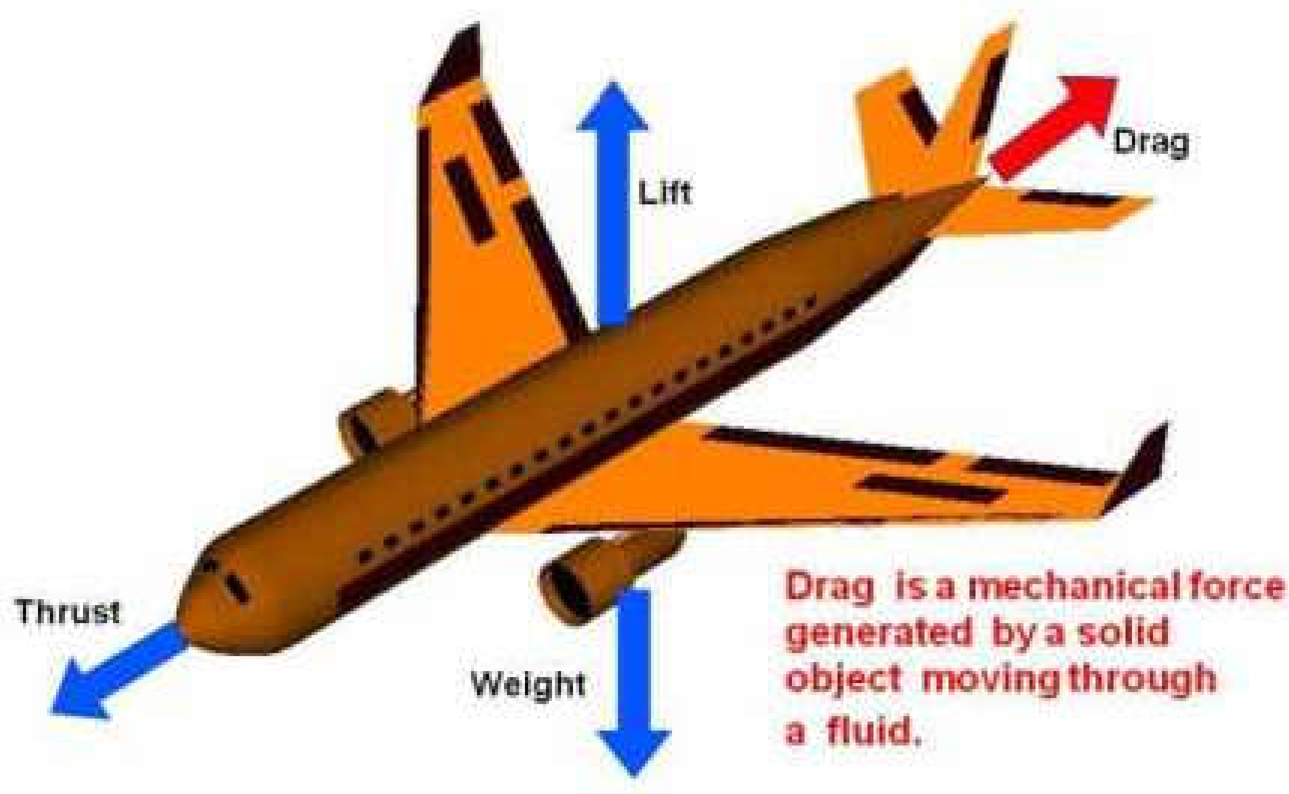
Fluid Resistance

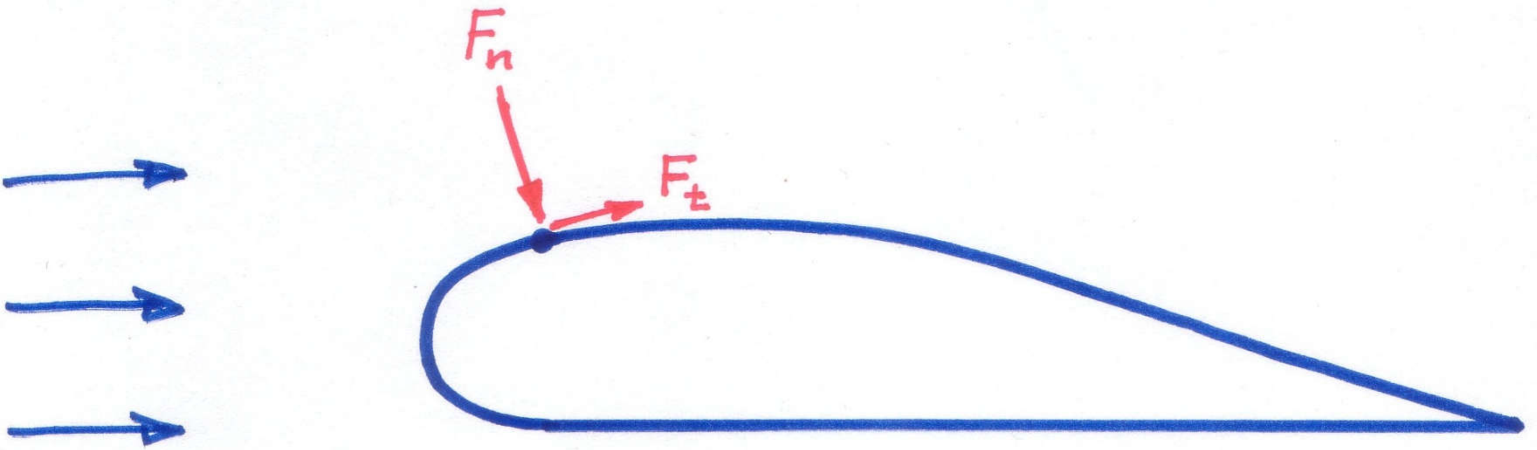
The transmission of energy from an object passing through a fluid to the fluid is known as **fluid resistance**.

The resistance of an object passing through a fluid increases as the speed of the object increases and as the viscosity of the fluid increases.

Drag

- Is the resistance an airplane experiences in moving forward through the air

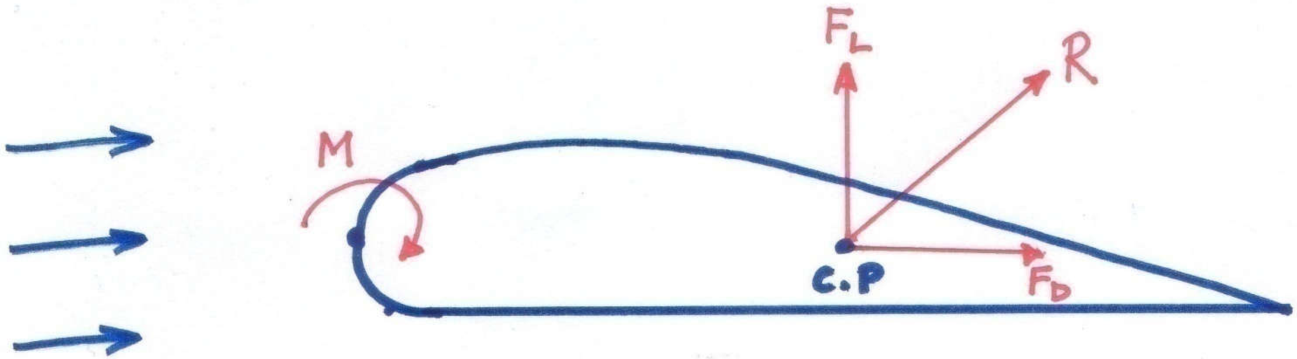




At any point on surface: $F_n = P\delta A \quad P: pressure$

$F_t = \tau\delta A \quad \tau: shear\ stress$

Integrate pressure and shear stress distributions around body surface



Drag F_D - component of resultant force in direction of flow

Lift F_L - component of resultant force perpendicular to direction of flow

Concept of Drag

Drag is the retarding force exerted on a moving body in a fluid medium

It does not attempt to turn the object, simply to slow it down

It is a function of the speed of the body, the size (and shape) of the body, and the fluid through which it is moving

Drag Force Due to Air

The drag force due to wind (air) acting on an object can be found by:

$$F_D = \frac{1}{2} \rho C_D V^2 A$$

where: F_D = drag force (N)

C_D = drag coefficient (no units)

V = velocity of object (m/s)

A = projected area (m^2)

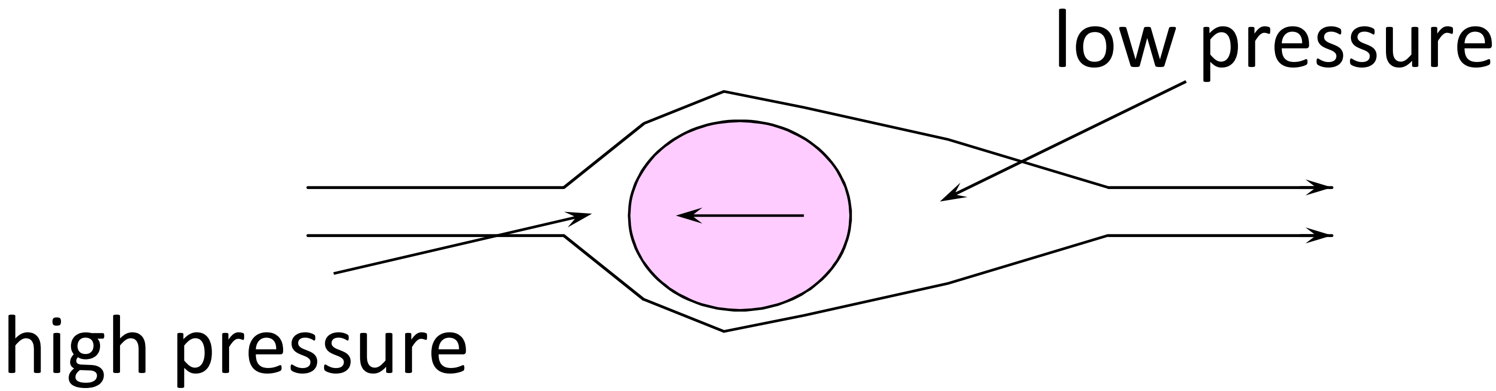
ρ = density of air (kg/m^3) {1.2 kg/m^3}

Surface and Form Drag

Surface drag is a result of the friction between the surface and the fluid.

The fluid closest to the object (**boundary layer**) rubs against the object creating friction.

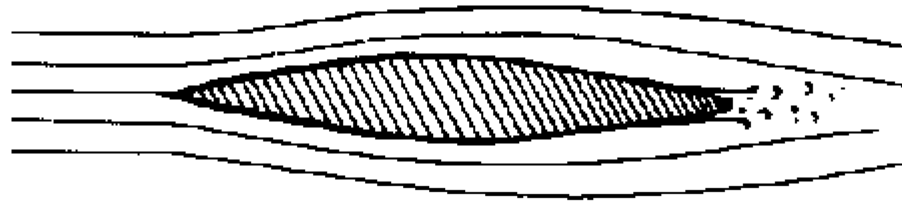
Form drag occurs when air is driven past an object and is diverted outward creating a low pressure region behind the object.

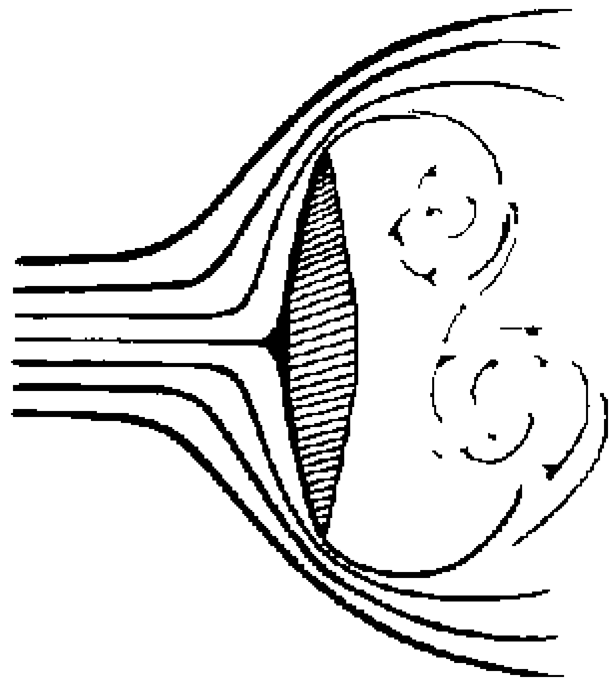


Form Drag

The orientation of the object will affect the frontal area and will play an important role in the amount of form drag.

Low form drag





High form drag

Lift and Drag

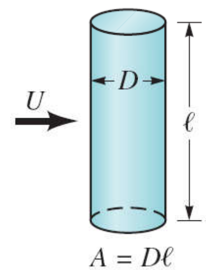
shear stress and pressure integrated over the surface of a body create force

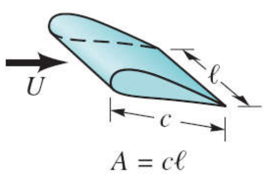
drag: force component in the direction of upstream velocity

lift: force normal to upstream velocity (might have 2 components in general case)

$$D = \int dF_x = \int p\cos\theta dA + \int \tau_w \sin\theta dA \qquad C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$

$$L = \int dF_y = \int p\sin\theta dA + \int \tau_w \cos\theta dA \qquad C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$





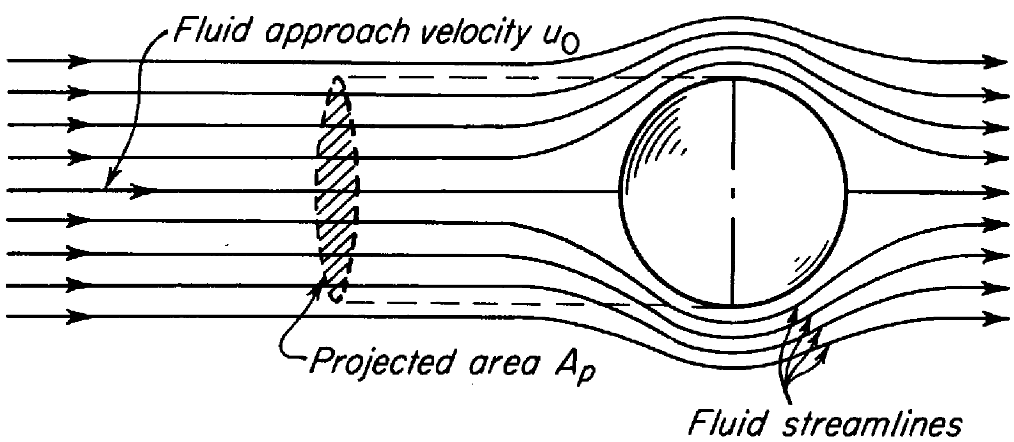
$$C_D = F_D/(^1/_2\, \rho U^2 A) = C_{D,pressure} + C_{D,friction}$$

Projected Area

The projected area used in the F_D is the area “seen” by the fluid.

Spherical Particle

$$A = \pi R^2 = \frac{\pi D^2}{4}$$

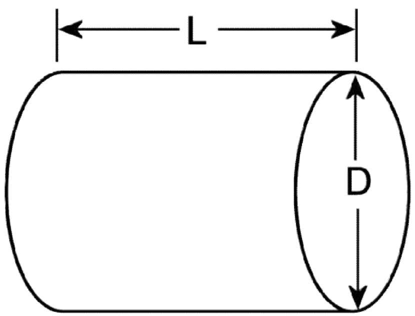


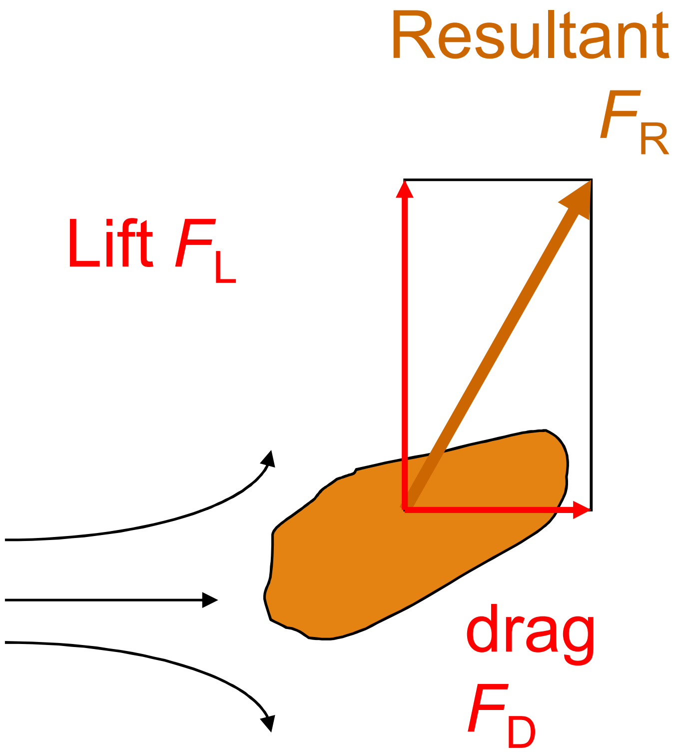
Projected Area

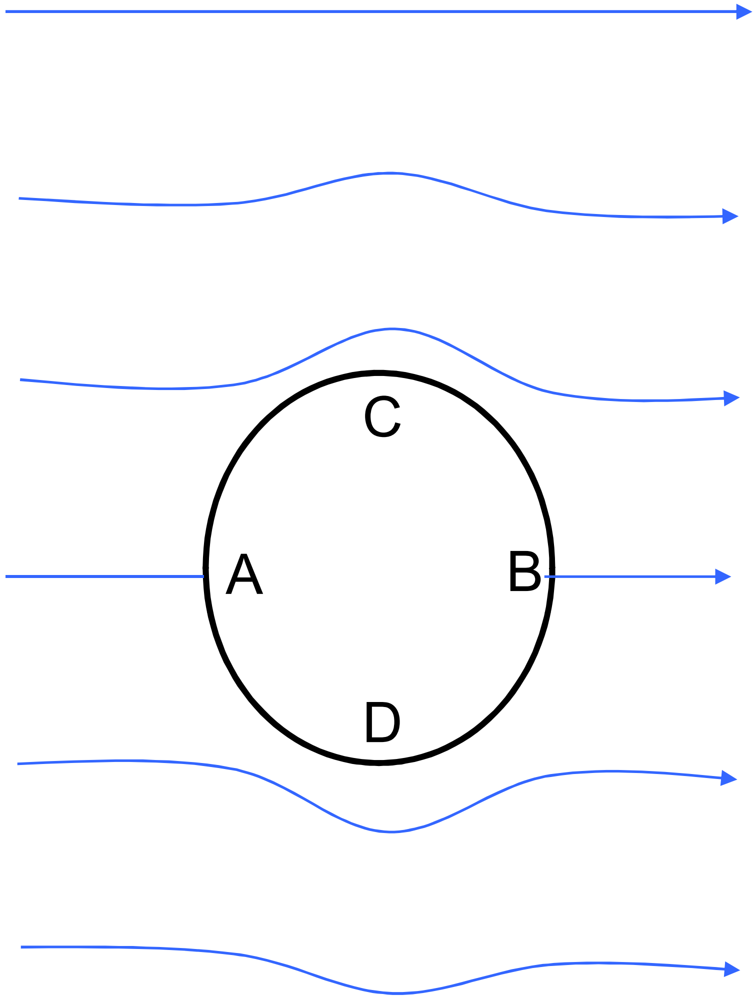
For objects having shapes other than spherical, it is necessary to specify the size, geometry and orientation **relative** to the direction of flow. The **direction of flow**.

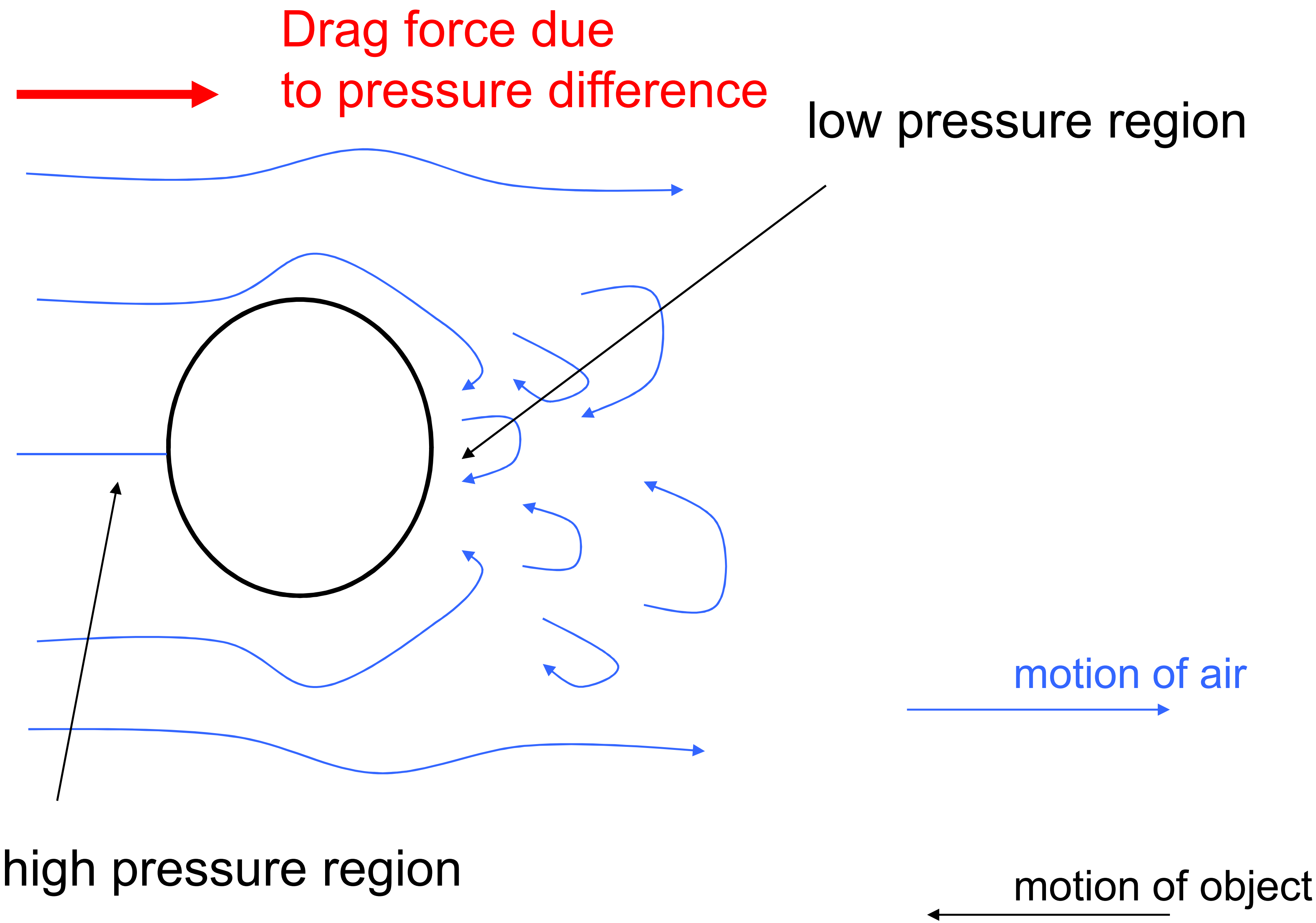
Cylinder

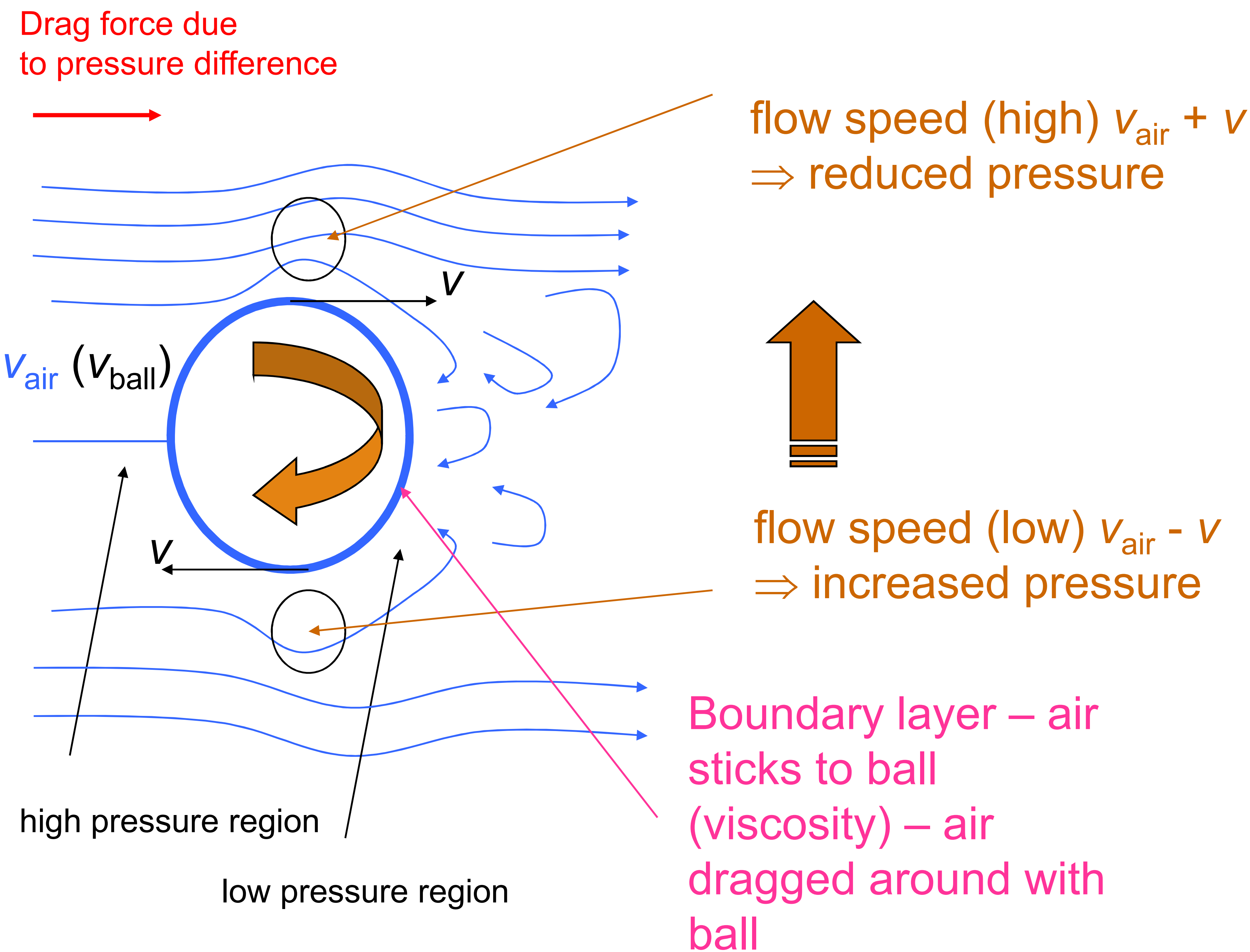
Axis perpendicular to flow	Rectangle	$A = LD$
Axis parallel to flow	Circle	$A = \dfrac{\pi D^2}{4}$



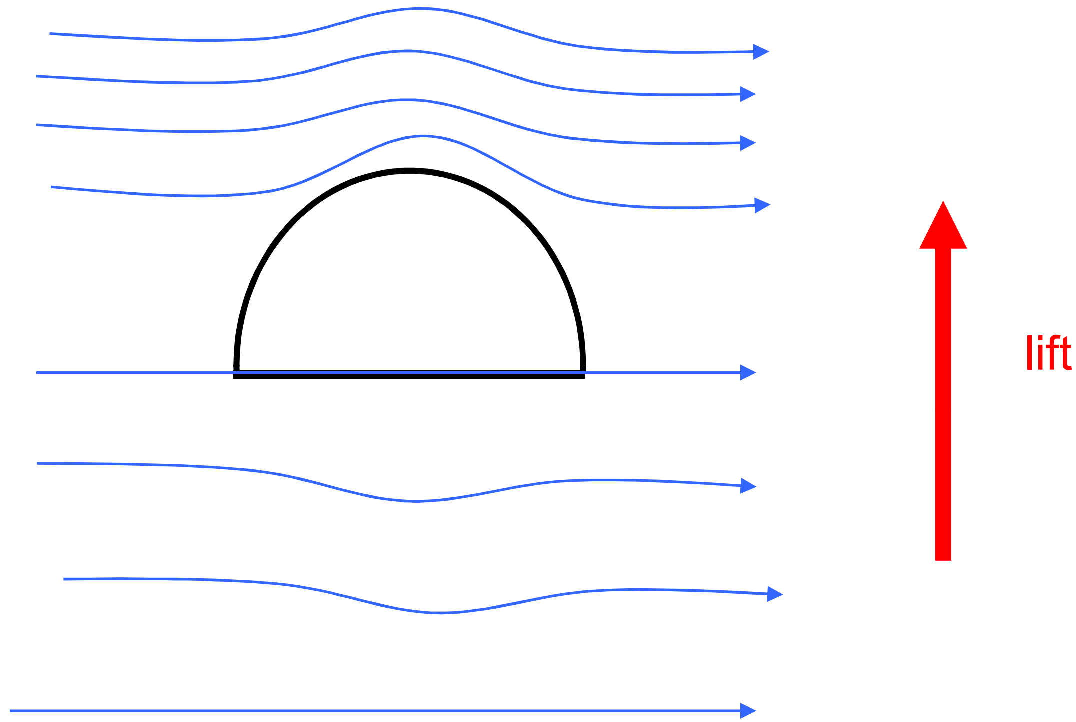

Resultant
F_R
Lift F_L
drag
F_D


C
A
B
D


Drag force due
to pressure difference
low pressure region
high pressure region
motion of air
motion of object



Drag force due
to pressure difference
flow speed (high) $v_{air} + v$
$\Rightarrow$ reduced pressure
v
v_{air} (v_{ball})
v
flow speed (low) $v_{air} - v$
$\Rightarrow$ increased pressure
Boundary layer – air sticks to ball (viscosity) – air dragged around with ball
high pressure region
low pressure region


lift

Drag Coefficient

For slow flow around a sphere and Re <10

$$C_d = \frac{24}{Re} = \frac{24\mu}{Du_0\rho}$$

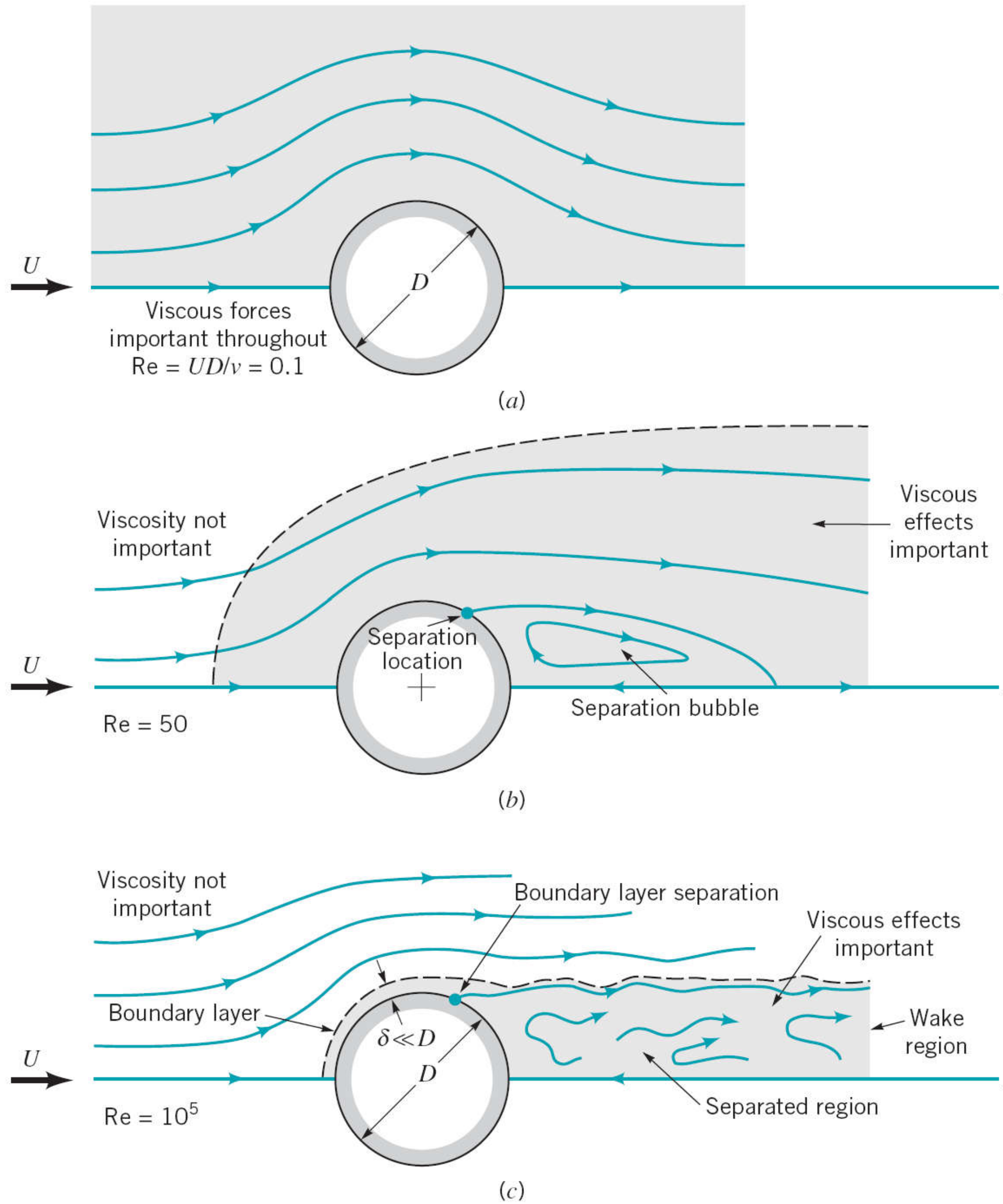
Recall: $$F_D = \frac{C_d A\rho u_0^2}{2}$$

Stokes' Law for Creeping Flow Around Sphere

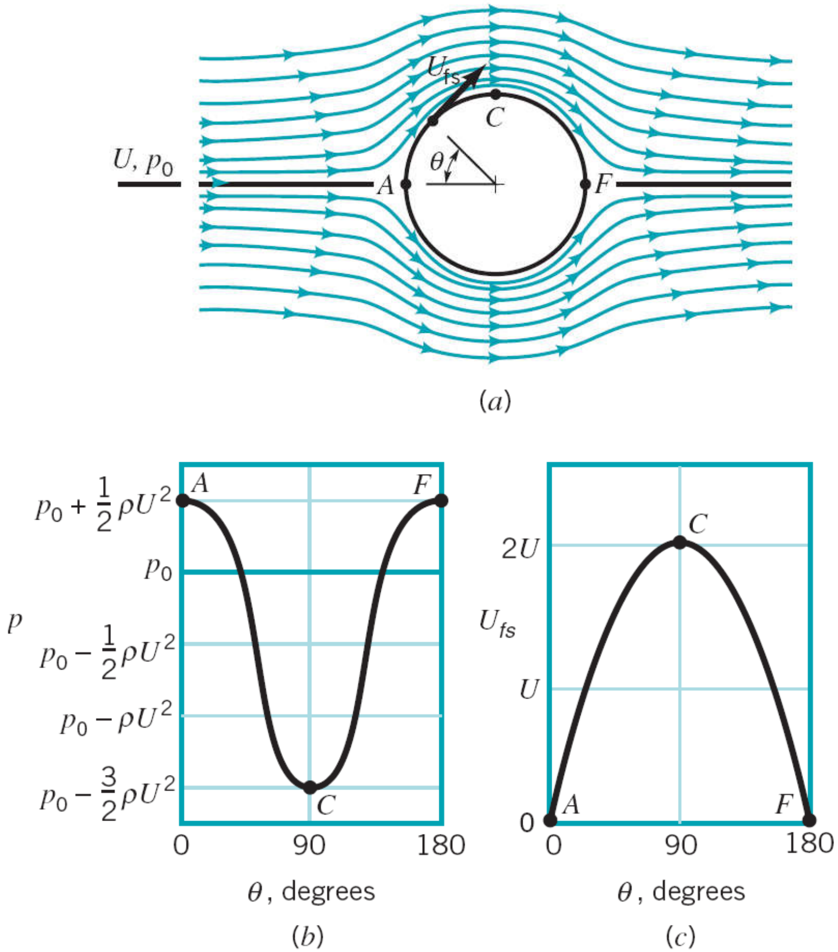
$$F_D = 3\pi\mu D u_0$$

Flow past an object

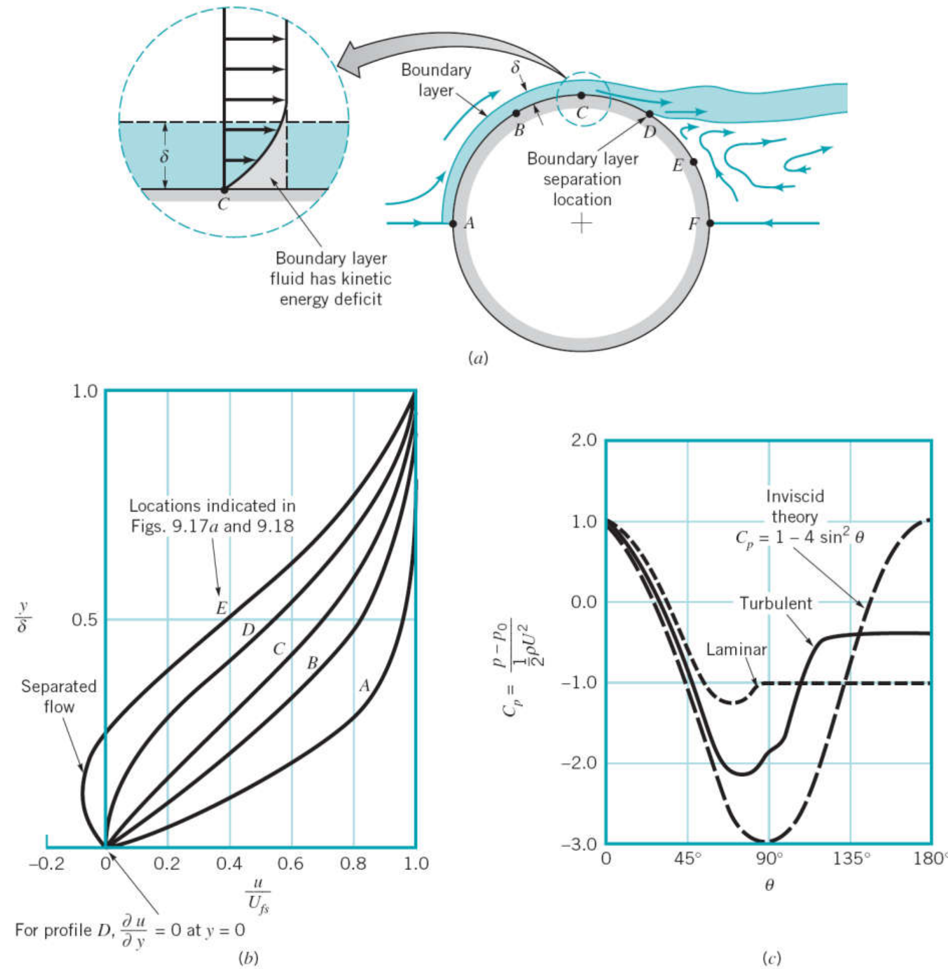
Character of the steady, viscous flow past a circular cylinder: **(a)** low Reynolds number flow, **(b)** moderate Reynolds number flow, **(c)** large Reynolds number flow.



Effect of pressure gradient

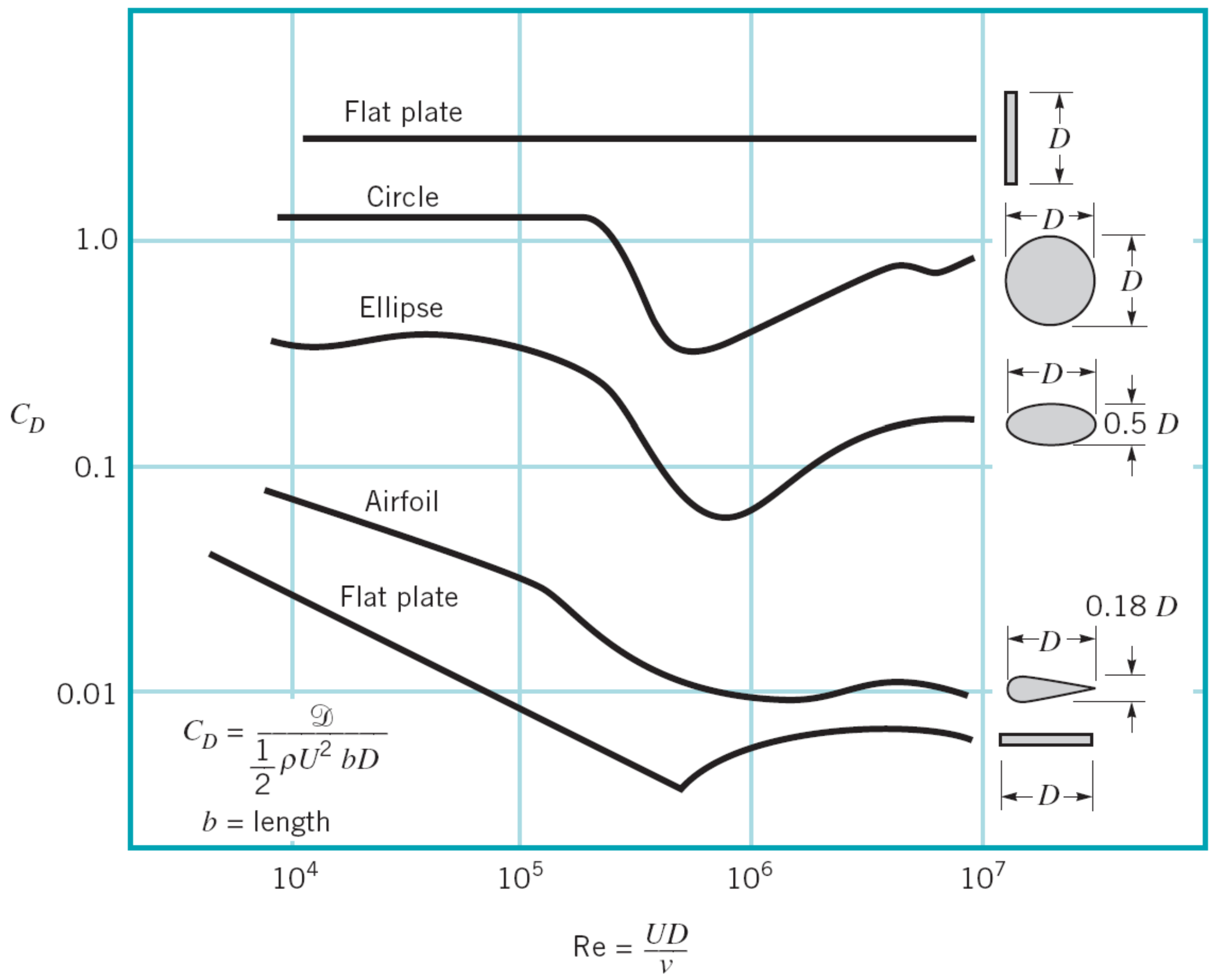


inviscid flow

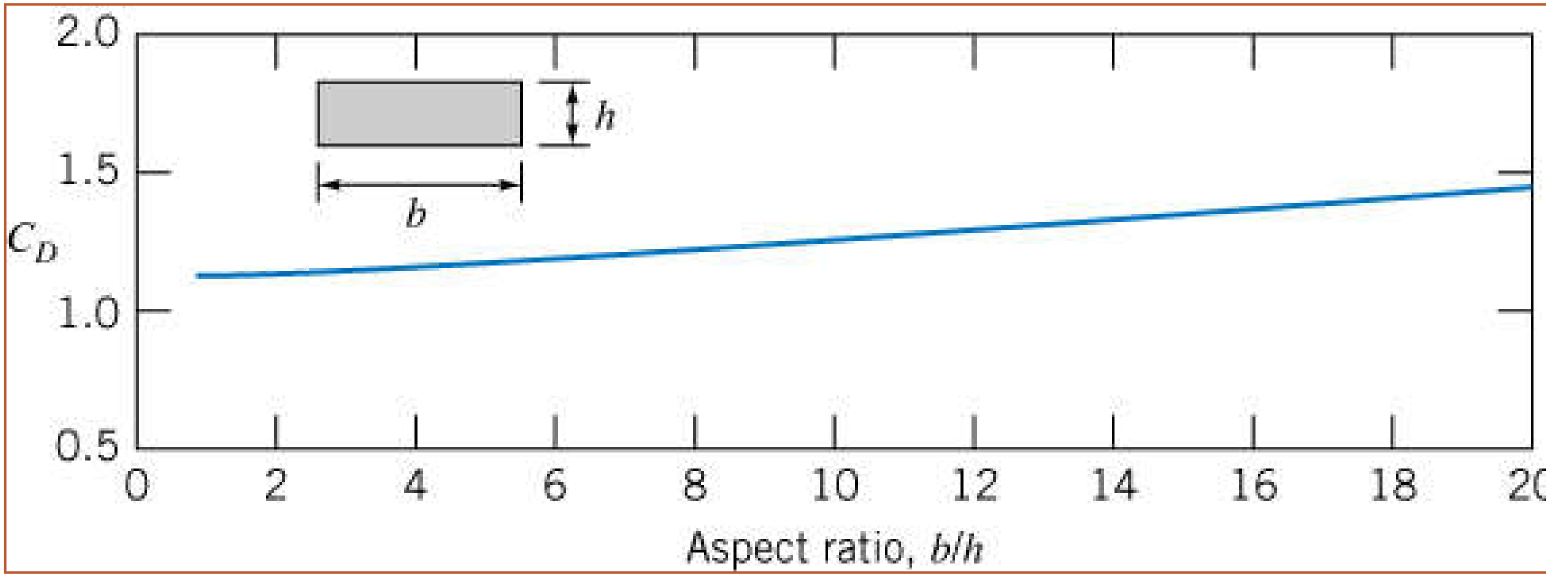


viscous flow

Examples



Flat plate
Circle
Ellipse
Airfoil
Flat plate
1.0
0.1
0.01
C_D
$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2 bD}$
b = length
10^4
10^5
10^6
10^7
Re = $\frac{UD}{\nu}$
D
0.5 D
0.18 D



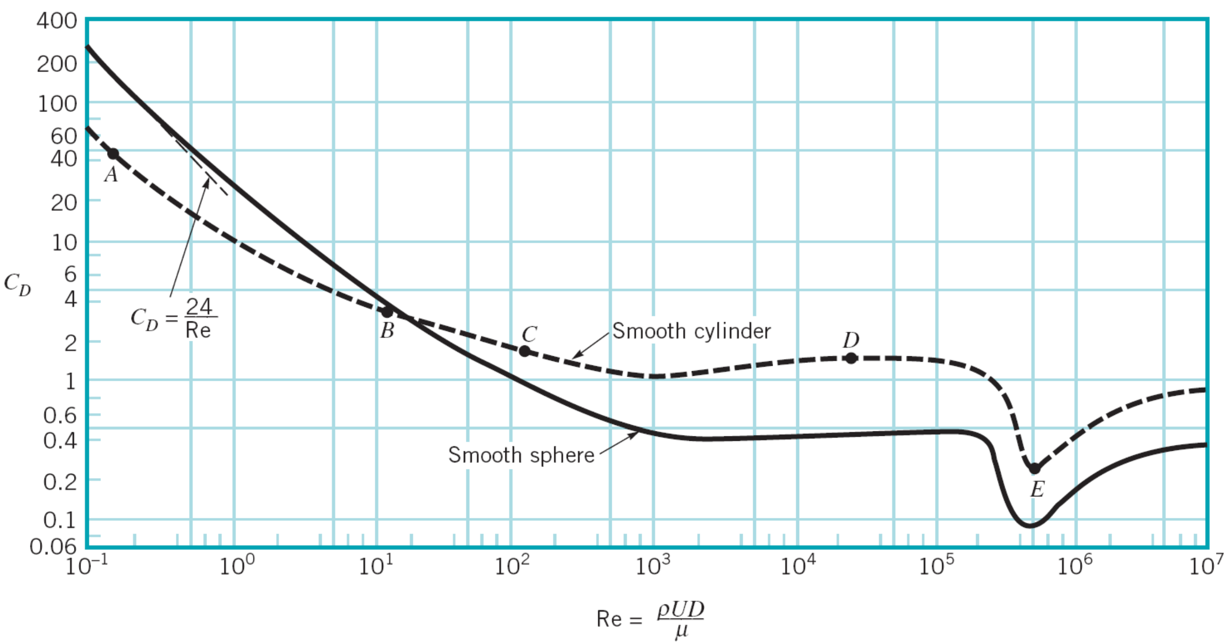
- b/h =1 square, C_D = 1.18; (disk; C_D = 1.17)
- C_D independent of Re for Re > 1000

Question: $C_D = F_D/(^1/_2\, \rho U^2 A)$

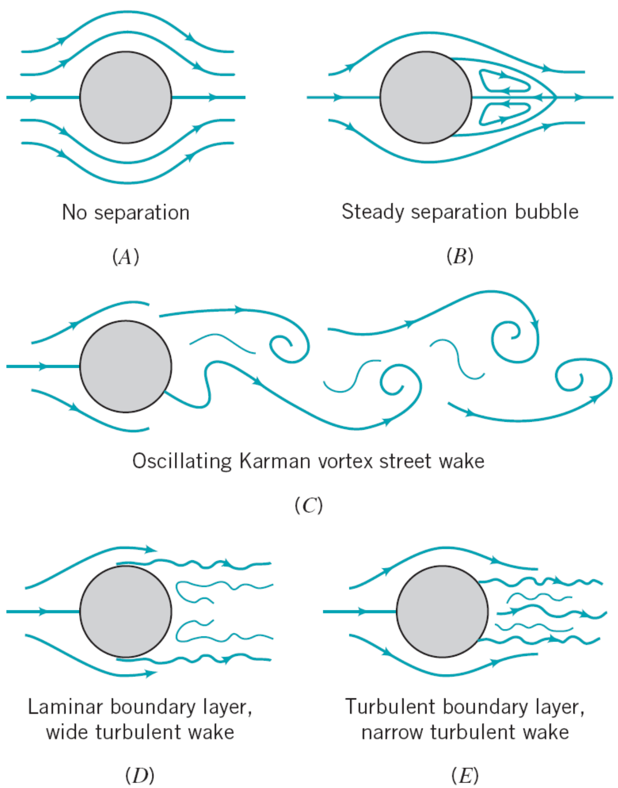
What happens to C_D if double area (b/h→2b/2h)?

What happens to F_D if double area (b/h→ 2b/2h)?

Drag dependence



(a)

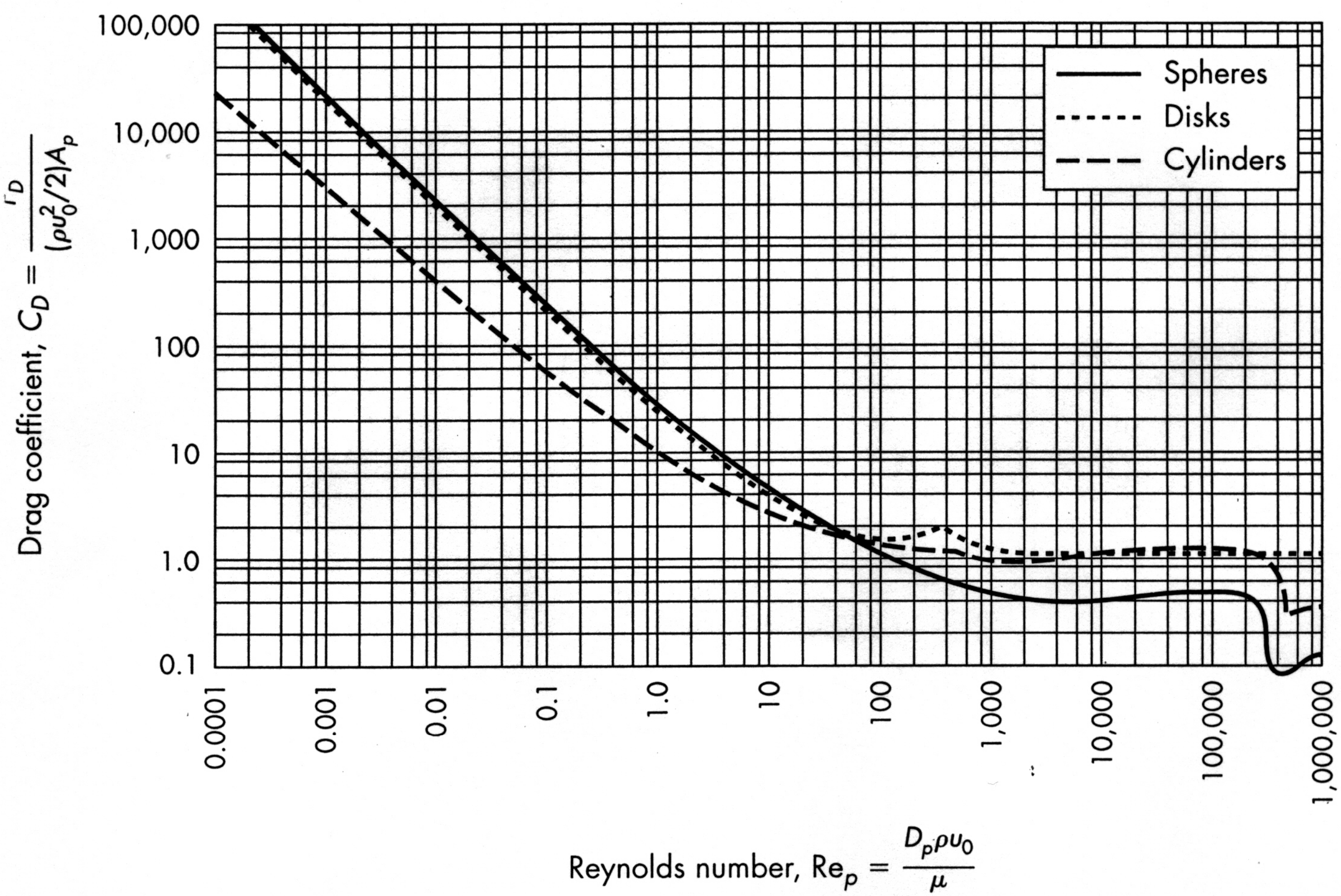


(b)

Drag Coefficient

$Re < 10 \quad C_d = 24/Re$

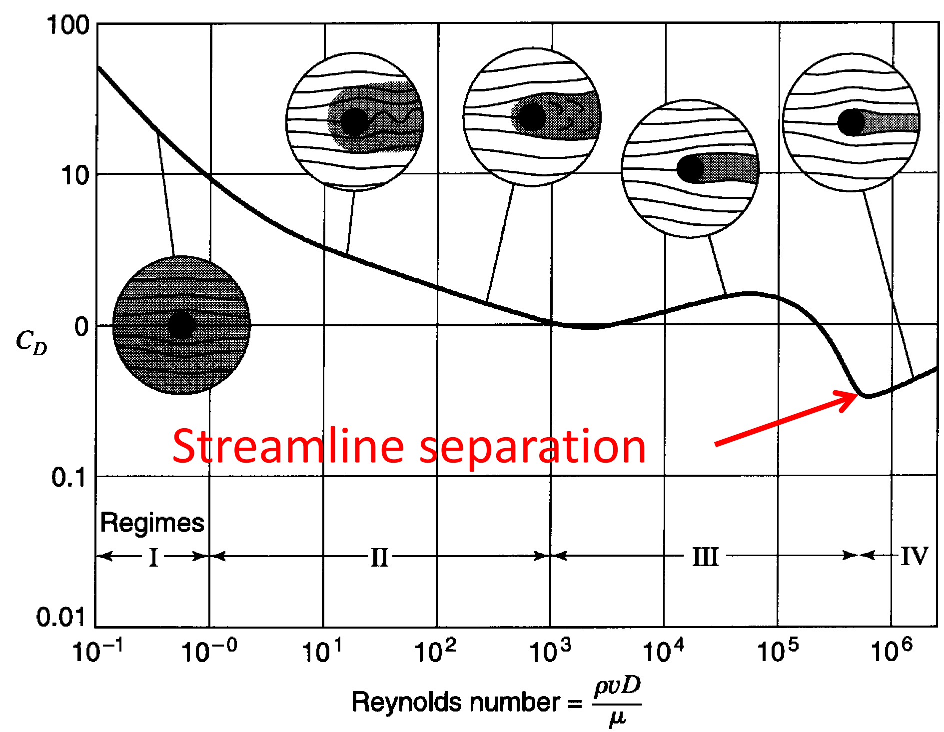
$Re > 1000 \quad C_d = 0.44$



for external flow: Re > 100 dominated by inertia, Re < 1 – by viscosity

Why Different Regions?

As the flow rate increases wake drag becomes an important factor. The streamline pattern becomes mixed at the rear of the particle thus causing a greater pressure at the front of the particle and thus an extra force term due to pressure difference. At very high Reynolds numbers completely separate in the wake.

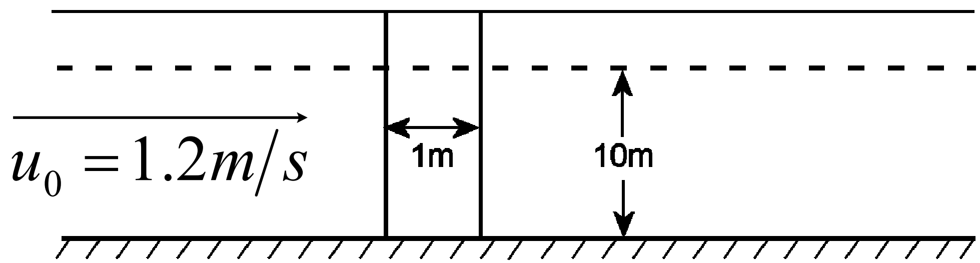


Example

A cylindrical bridge pier 1 meter in diameter is submerged to a depth of 10m in a river at 20°C. Water is flowing past at a velocity of 1.2 m/s. Calculate the force in Newtons on the pier.

$$\rho_{water} = 998.2 kg/m^3$$

$$\mu_{water} = 1.005 \times x10^{-3} kg/m \cdot s$$



$$F_k = \frac{C_d A \rho u_0^2}{2}$$

$$Re = \frac{\rho u_0 D}{\mu} = \frac{998.2\, kg/m^3 \times 1.2\, m/s \times 1m}{1.005 \times 10^{-3}\, kg/m \cdot s} = 1.192 \times 10^6$$

From figure $C_d \approx 0.35$

Projected Area = DL = 10 m^2

$$F_k = \frac{0.35}{2} \times 10m^2 \times 998.2 \frac{kg}{m^3} \times (1.2)^2 \frac{m^2}{s^2} = 2{,}515N$$

Experiments were conducted in a wind tunnel with a wind speed of 50km/hr on a flat plate of size 2m long and 1m wide. The density of air is 1.15kg/m^3. The coefficient of lift and drag are 0.75 and 0.15 respectively. Determine:

(i) Lift force.

(ii) Drag force.

(iii) Resultant force. And

(iv) Direction of resultant force.

(v) The power exerted by air on the plate.

Sol:

Areaof the plate, A=2×1 = $2m^2$.

Velocity of air, U = 50km/hr = $\frac{50\times1000}{60\times60}$ = 13.89m/s.

Density of air, ρ = 1.15kg/m^3.

Coefficient of drag, C_D= 0.15

Coefficient of drag, C_L= 0.75

(i)Lift force (F_L)

$F_L = C_L A\times\frac{\rho U^2}{2}$

$= 0.75\times2\times\frac{1.15\times13.89^2}{2}$N

F_L=166.404N

(ii)Drag force (F_D)

$F_D = C_D A\times\frac{\rho U^2}{2}$

$= 0.15\times2\times\frac{1.15\times13.89^2}{2}$N

F_D=33.28N

(iii)Resultant force (F_R)

$F_R = \sqrt{{F_D}^2 + {F_L}^2}$

$= \sqrt{(33.28)^2 + (166.404)^2}$

F_R = 169.69N

(iii)The direction of Resultant force (θ):

The direction of resultant force is given by

$\tan\theta = \frac{F_L}{F_D} = \frac{166.404}{33.28} = 5.0$

$\theta = tan^{-1}(5.0)$

$\theta = 78.69^0$

Power exerted by air on the plate

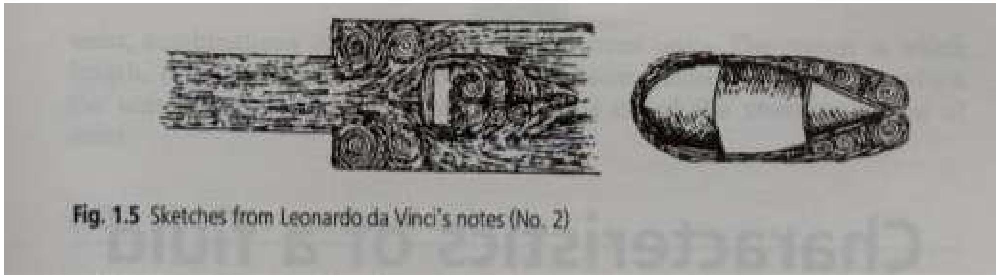
Power = Force in the direction of motion × velocity

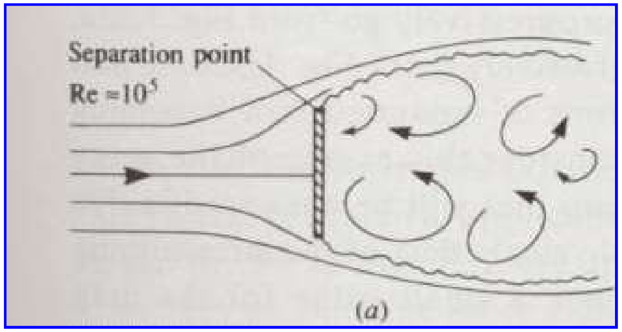
= $F_D\times$U Nm/s

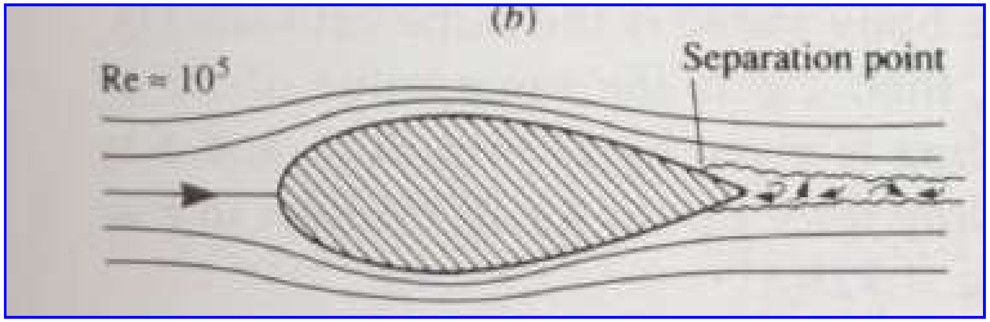
= 33.28 × 13.89 W (watt = Nm/s)

STREAMLINING

Streamlining is the attempt to reduce the drag on a body







C_D ~ 2 for flat plate

C_D ~ 0.06

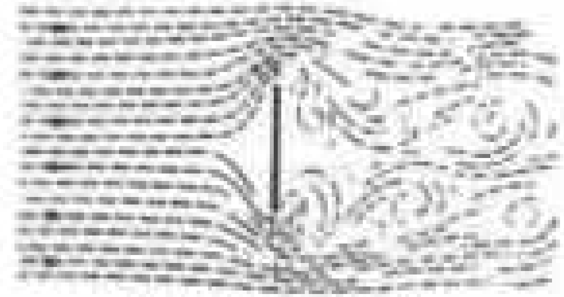
Streamlining

The less drag you have…

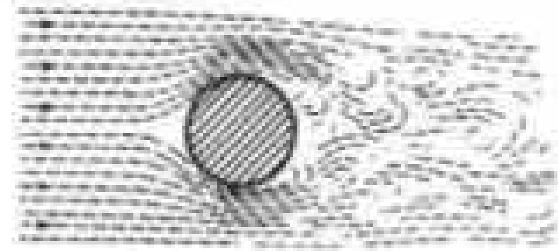
- Flying a glider: the further you can fly
- Flying an airplane: the less fuel you use

Therefore streamlining is important

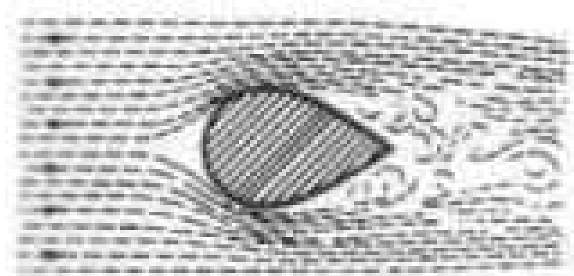
- A design device by which a body is shaped to minimize drag



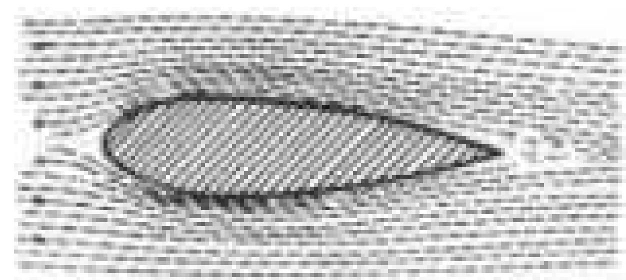
Resistance, 100%



Resistance, 50%



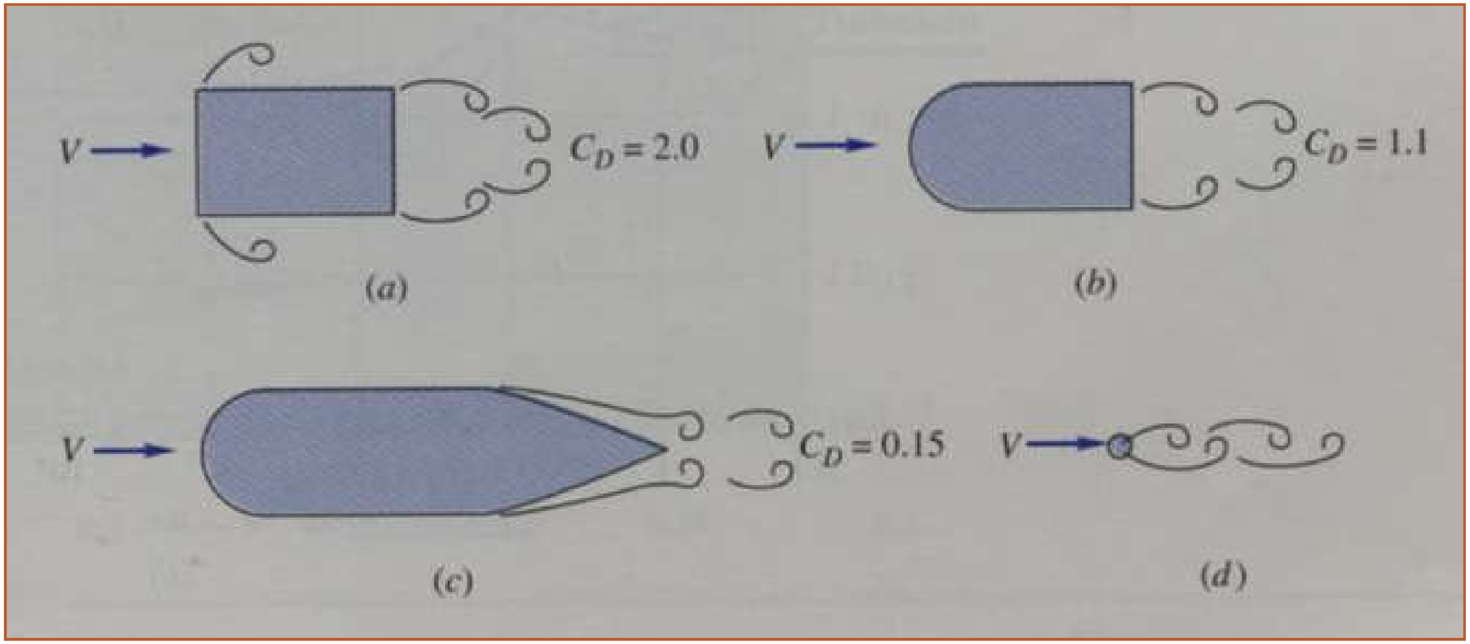
Resistance, 15%



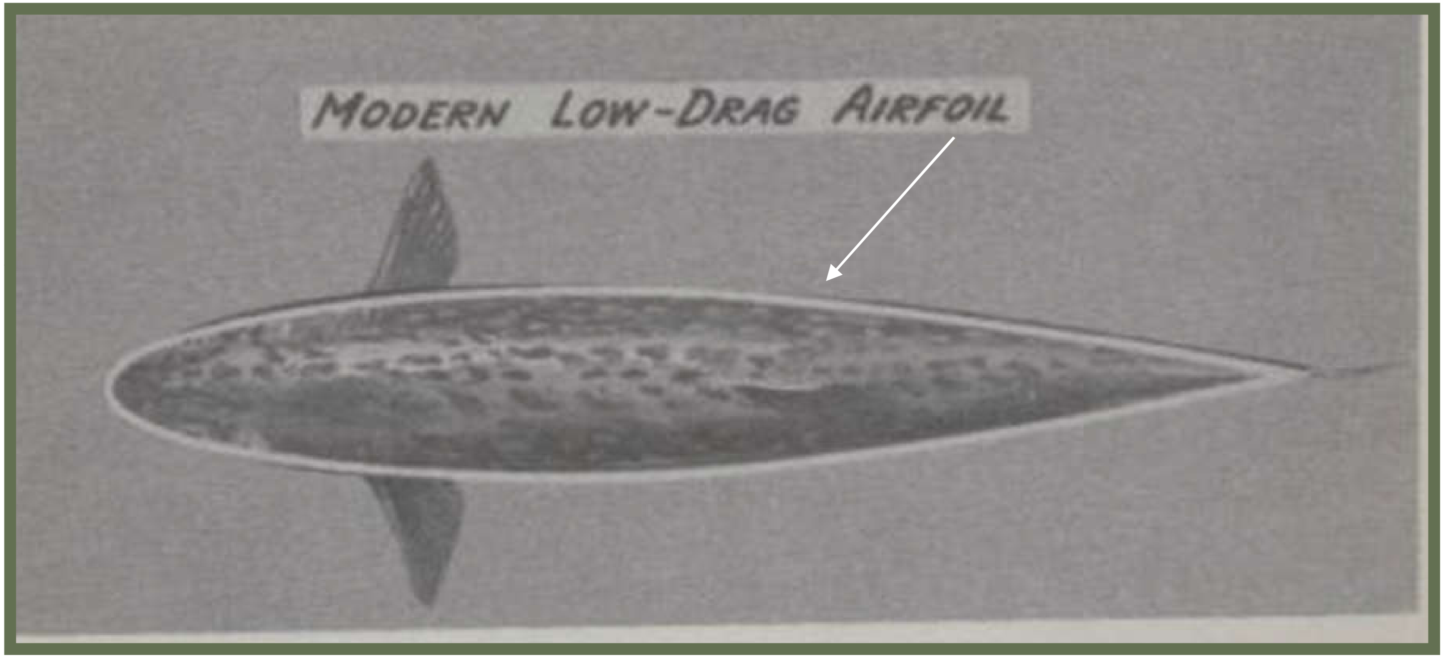
Resistance, 5%

In general, the importance of streamlining to reduce drag.

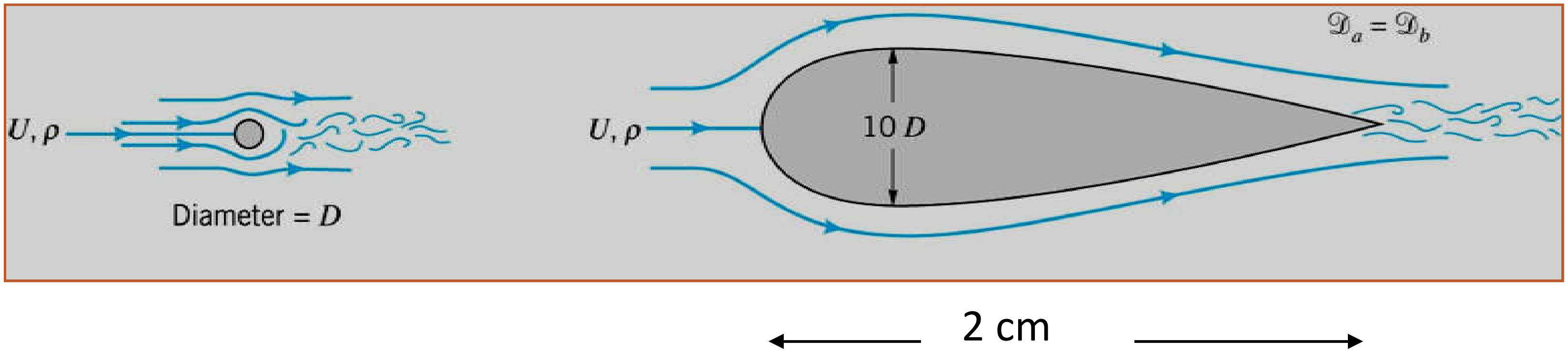
2-D rectangular cylinder



STREAMLINING



STREAMLINING



~ same drag AND wake

Non-circular Channels

Equivalent diameter defined as 4 times the hydraulic radius (r_H).

$$r_H = \frac{A}{L_p}$$

Where, A = cross-sectional area of channel

L_p = perimeter of channel in contact with fluid

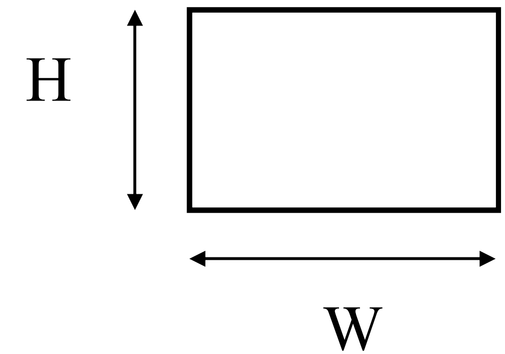
Hydraulic radius of circular tube,

$$r_H = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

The equivalent diameter is **4 r_H**.

For a rectangular duct with width W and height H, the hydraulic diameter is

$$D_h = \frac{4A}{P} = \frac{4WH}{2(W+H)} = \frac{2WH}{W+H}$$



Annulus between two circular pipes

$$r_H = \frac{\pi \frac{D_0^2}{4} - \pi \frac{D_i^2}{4}}{\pi D_i + \pi D_0} = \frac{D_0 - D_i}{4}$$

Sphericity

Surface area of sphere, $S_p = 4\,\pi\, r^2 = \pi\, D_p^2$

Volume of sphere, $V_p = (4/3)\,\pi\, r^3 = (1/6)\,\pi\, D_p^3$

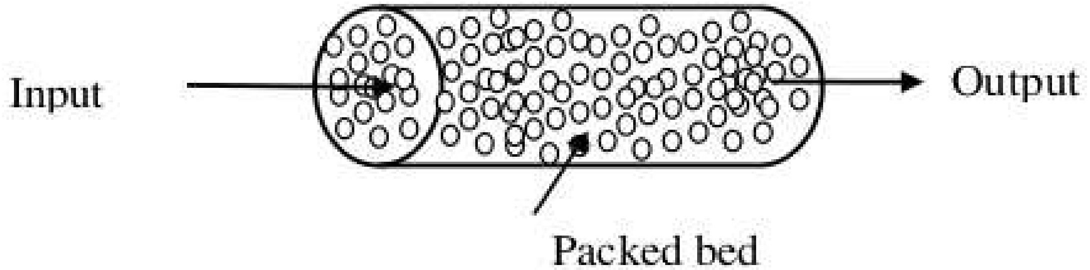
Sphericity (ϕ_s) : The surface-volume ratio for a sphere of diameter D_p **divided** by the surface-volume ratio for the particle whose Nominal size is D_p.

$$\phi_s = \frac{6/D_p}{S_p/v_p}$$

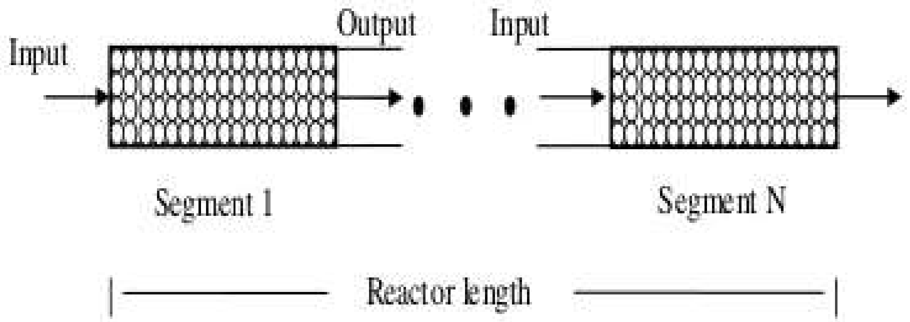
$$\frac{S_p}{V_p} = \frac{6}{\phi_s D_p}$$

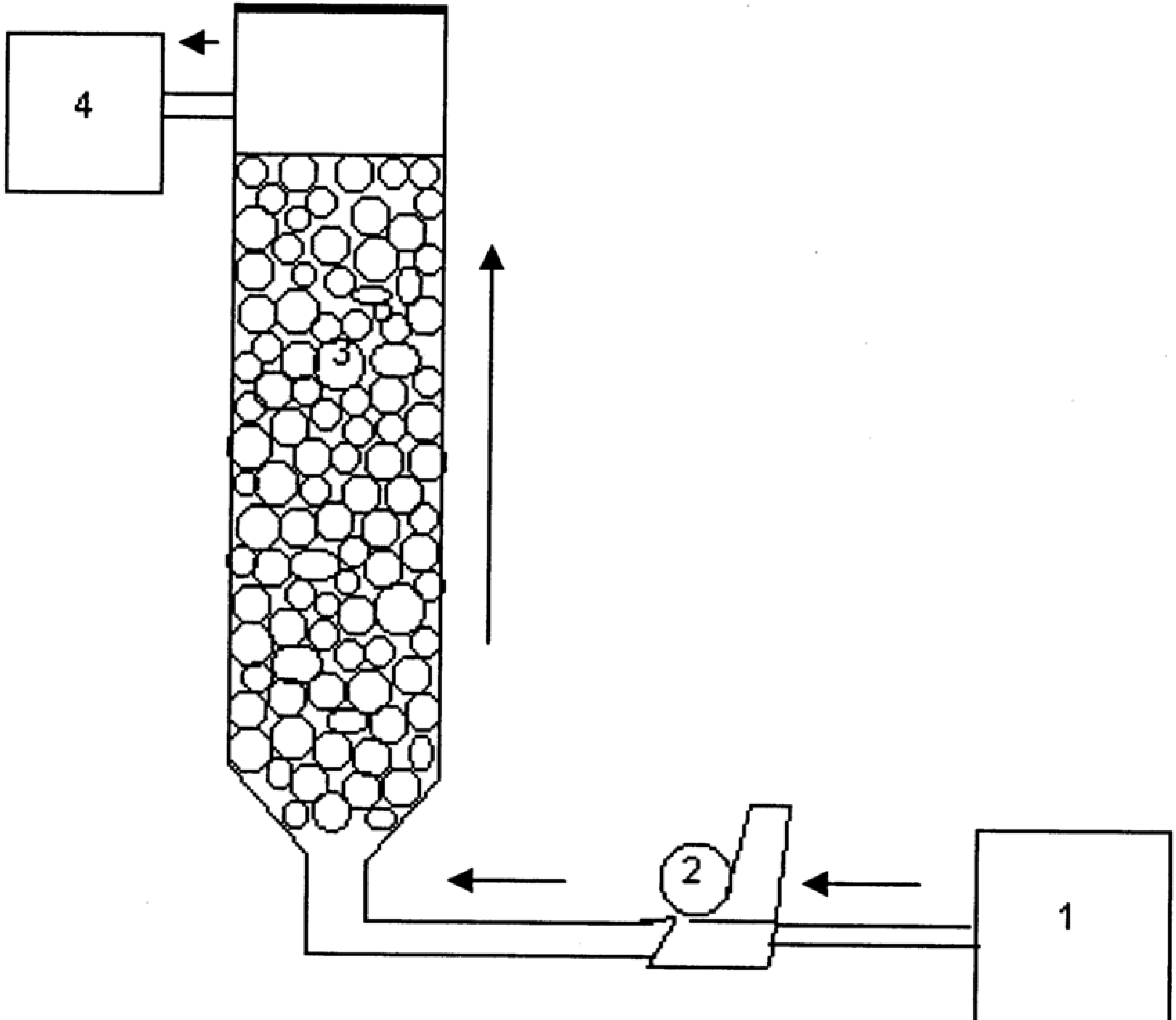
Geometry

Schematic of a Packed Bed Reactor



Segmented geometry





4
3
2
1

Advantages of Packed Bed Reactor

Higher conversion per unit mass of catalyst than other catalytic reactors.

Continuous operation

No moving parts to wear out.

Low operating cost

Catalyst stays in the reactor

Reaction mixture/catalyst separation is easy.

Effective at high temperatures and pressures

FLOW IN PACKED BEDS

FLUID FRICTION IN POROUS MEDIA

PACKED TOWERS

- Packed towers are finding applications in adsorption, absorption, ion-exchange, distillation, humidification, catalytic reactions, regenerative heaters etc.,
- Packing is to provide a good contact between the contacting phases.
- Based on the method of packing, Packings are classified as

  (a) Random packings

  (b) Stacked packings

2.Pressure drop

At a steady state, and negligible gravity effect,
The pressure drop is given by;

$$\Delta p = \frac{32\mu v\ \Delta L}{D^2} = \frac{32\mu(v'/\varepsilon)\ \Delta L}{(4r_H)^2} = \frac{(72)\mu v'\ \Delta L(1-\varepsilon)^2}{\varepsilon^3 D_p^2}$$

However, the experimental show that the constant should be 150, which gives the *Kozeny-Carman* equation for laminar flow, void fraction less than 0.5, effective particle diameter D_p and $N_{Re} < 10$

$$\Delta p = \frac{150\mu v'\ \Delta L}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

dP/dx = Pressure gradient

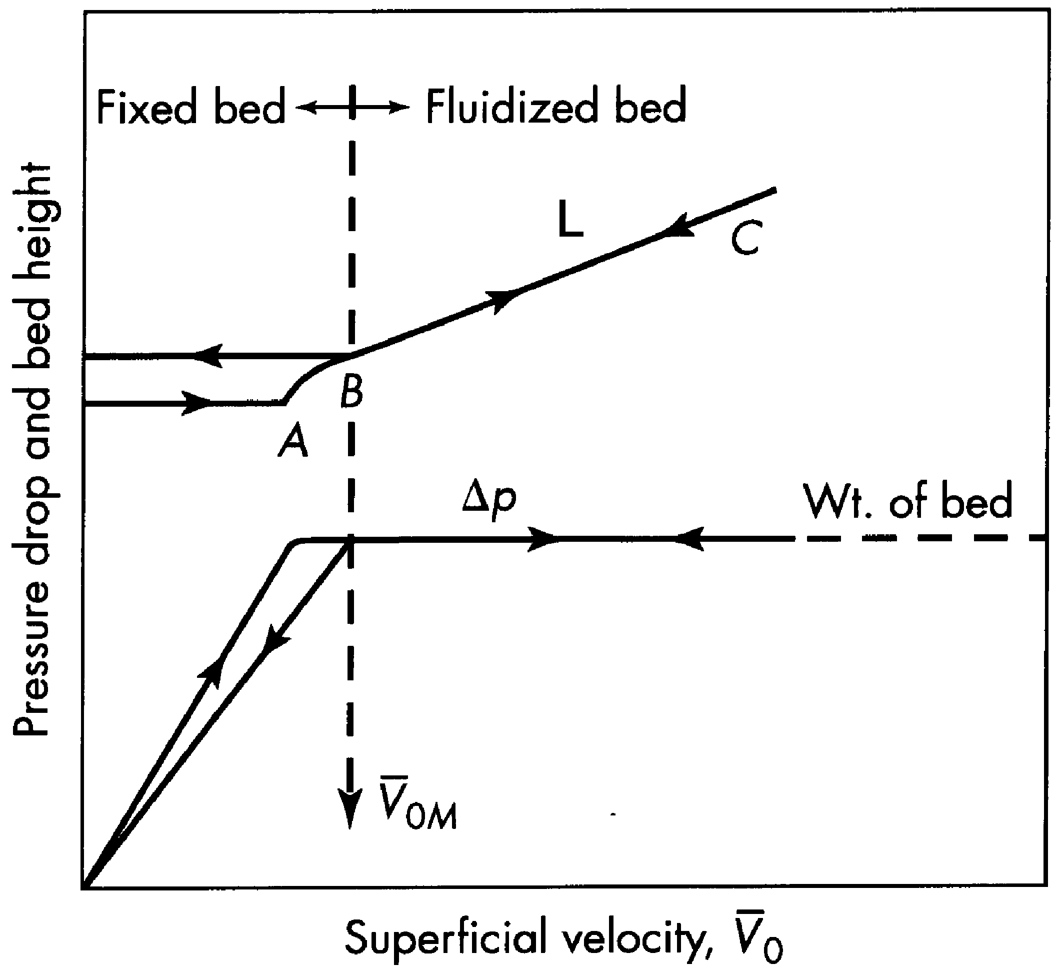
Φ = Sphericity (1 for perfect sphere)

D_p = Particle diameter (m)

ε = Porosity of the media

V = Velocity

Response to Superficial Velocities



Turbulent Flow

One cannot use the Hagen-Poiseuille approximation when flow is turbulent. After substituting in D_h and velocity correction

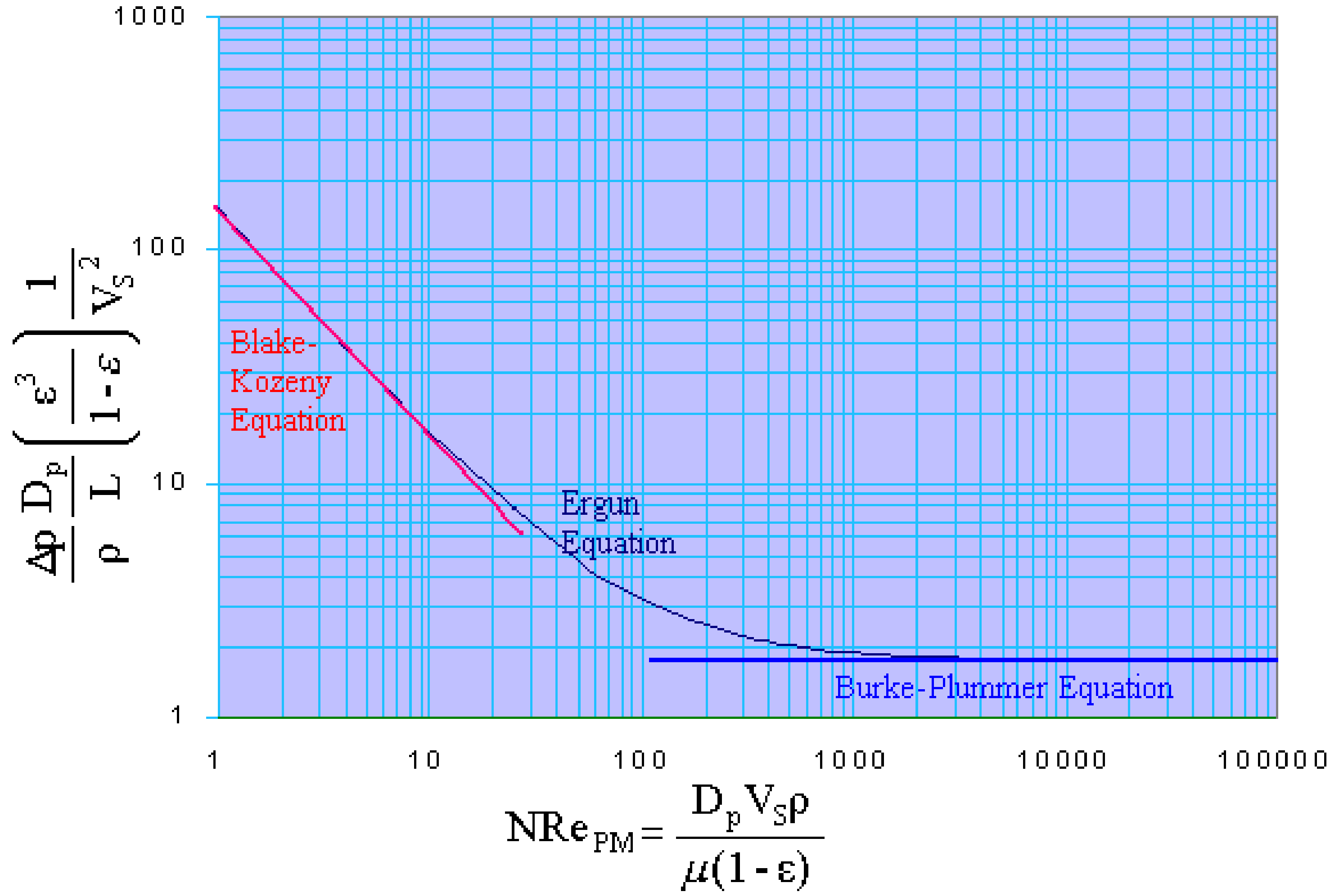
$$\Delta p = \frac{3 f \rho u_0^2 L}{D_p} \frac{(1-\varepsilon)}{\varepsilon^3}$$

Experimentally:

$$Re_p > 1{,}000$$

$$\Delta p = \frac{1.75 \rho u_0^2 L}{D_p} \frac{(1-\varepsilon)}{\varepsilon^3}$$

Burke-Plummer Equation



1000
100
10
1
1
10
100
1000
10000
100000
Blake-Kozeny Equation
Ergun Equation
Burke-Plummer Equation
$\frac{\Delta p}{\rho}\frac{D_p}{L}\left(\frac{\varepsilon^3}{1-\varepsilon}\right)\frac{1}{V_S^2}$
$NRe_{PM} = \frac{D_p V_S \rho}{\mu(1-\varepsilon)}$

Intermediate Flow

$$\Delta p = \frac{150 \mu u_0 L_b}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75 \rho u_0^2 L_b}{D_p} \frac{(1-\varepsilon)}{\varepsilon^3}$$

Ergun Equation

Irregular Shapes

So the final Ergun equation is:

$$\Delta p = \frac{150 \mu u_0 L_b}{\Phi_s^2 D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75 \rho u_0^2 L_b}{\Phi_s D_p} \frac{(1-\varepsilon)}{\varepsilon^3}$$

Problem

Air (ρ = 1.22 Kg/m^3, μ = 1.9 X 10 $^{--5}$ pa.s) is flowing in a fixed bed of a diameter 0.5 m and height 2.5 m. The bed is packed with spherical particles of diameter 10 mm. The void fraction is 0.38. The air mass flow rate is 0.5 kg/s. Calculate the pressure drop across the bed of particles.

Solution

Q = volumetric flow rate $= \frac{0.5}{1.22} = 0.41 \ m^3/s$

$$A = \frac{\pi}{4} D^2 = \left(\frac{\pi}{4}\right)(0.5)^2 = 0.1963 \ m^2$$

$$u_{\infty} = \frac{Q}{A} = \frac{0.41}{0.1963} = 2.1 \ m/s$$

$$Re_p = \frac{\rho u_{\infty} D_p}{(1-\varepsilon)\mu} = \frac{(1.22)(2.1)(10 X 10^{-3})}{(1-0.38) \ (1.9 X 10^{-5})}$$

$$Re_p = 2174$$

$$f_p = \frac{150}{2174} + 1.75 = 1.819 = \frac{D_p \varepsilon^3}{\rho u_{\infty}^2 (1-\varepsilon)} \frac{|\Delta P|}{L}$$

$$\Delta P = \frac{(1.819)(1.22)(2.1)^2(1-0.38)(2.5)}{(10 X 10^{-3})(0.38)^3}$$ **= 0.276 x 10^5 pa.**