Fluid as a continuum

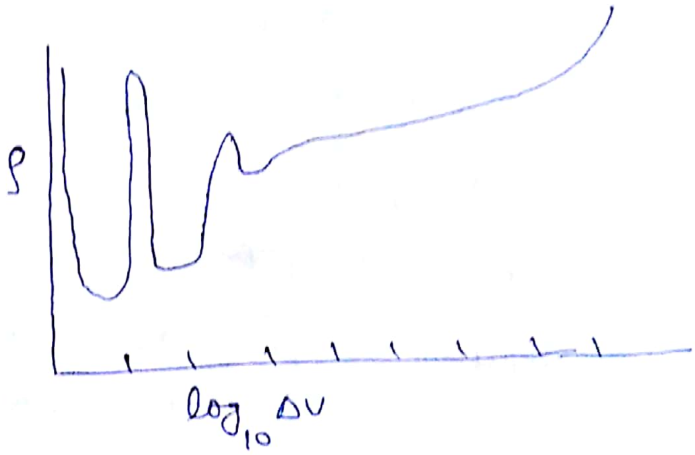
Concept of a Continuum

In the mathematical description of fluid flow, it is necessary to assume that the flow quantities such as velocity and pressure and fluid properties vary continuously from one point to another.

Consider the variation of density as a function of the size of an element ΔV. Density at a point is defined as

$$\rho = \lim_{\Delta V \to 0} \frac{m}{\Delta V}$$

where m is the mass of the element constructed around the point of interest. Figure shows a variation of ρ as a function of ΔV.



If ΔV is very large, ρ is affected by the inhomogeneities in fluid itself arising from varying composition and temperature distribution.

As ΔV becomes smaller, an almost uniform density is reached, independent of ΔV.

If ΔV is very small, random fluctuations in position of atoms/molecules would change their number (and hence m) from one instant to another. If ΔV were to be so small as to cover just one particle, the density would fluctuate b/w zero and a finite value, depending on whether or not the particle found in ΔV at a given instant. In the continuum approximation, the point density is ~~[illegible]~~ defined as that value of ρ which occurs at the smallest magnitude of ΔV, before, statistical fluctuations become significant. ΔV_c, at which this limit process is carried out is called the continuum limit. The definition of density now becomes

$$\rho = \lim_{\Delta V \to \Delta V_c} \frac{m}{\Delta V}$$

Similarly, the magnitude of a point stress component is defined as

$$\sigma = \lim_{\Delta A \to \Delta A_c} \frac{F}{\Delta A}$$

where F is the associated force quantity and ΔA is the infinitesimal area on which it acts. The <u>local velocity</u> of is the velocity of a closed surface constructed around the point of interest. This surface is small enough for velocity to be sensibly uniform within but large enough to have a sufficiently large population of elementary particles.

The continuum approximation breaks down if the density of the fluid is so small that molecular motion occurs on the same scale as mean fluid movement. If λ is the mean free path of the molecules and L the characteristic dimension of macroscopic flow, the continuum approximation is valid if $\frac{\lambda}{L} << 1$ and not valid if $\frac{\lambda}{L} \sim 1$, order of unity.

The ratio λ/L is called Knudsen number. It is of order unity in rarefied gas flows, for example, in the upper layers of the earth's atmosphere.

The continuum approximation has wide applicability. Consider the following example. Under atmospheric conditions near the surface of the earth, 1 mm^3 vol. of air contains around 3×10^{16} molecules. This is a very large number. Over the length scales involved in engg. problems, 1 mm^3 is a small vol^m and yet it is sufficiently populated for continuum approxn to be valid.

Terminology in Fluid Dynamics

The concept of pressure is central to the study of both fluid statics and fluid dynamics. A pressure can be identified for every point in a body of fluid, regardless of whether the fluid is in motion or not. Pressure can be measured using an aneroid, Bourdon tube, mercury column, or various other methods.

Some of the terminology that is necessary in the study of fluid dynamics is not found in other similar areas of study. In particular, some of the terminology ~~that is necessary~~ in used in fluid dynamics is not used in fluid statics.

Terminology in incompressible fluid dynamics

The concept of total pressure and dynamic pressure arise from Bernoulli's eqn and are significant in the study of all fluid flows. (These two pressures are not pressures in the usual sense - they can not be measured using an aneroid, Bourdon tube or mercury column). To avoid potential ambiguity when referring to pressure in fluid dynamics, many authors use the term static pressure to distinguish it from total pressure and dynamic pressure. Static pressure is identical to pressure and can be identified for every point in a fluid flow field.

A point in a fluid flow where the flow has come to rest (i.e. speed is equal to zero adjacent to some solid body immersed in the fluid flow) is of special significance. It is of such importance that it is given a special name - a stagnation point. The static pressure at the stagnation point is of special significance and is given its own name— stagnation pressure. In incompressible flows, the stagnation pressure at a stagnation point is equal to the total pressure throughout the flow field.

Terminology in compressible fluid dynamics

In a compressible fluid, such as air, the temp. and density are essential when determining the state of the fluid. In addition to the concept of total pressure (also Known as stagnation pressure), the concepts of total (or stagnation) temp. and total (or stagnation) density are also essential in any study of compressible fluid flows.

Two branches of Mechanics

- **kinematics**: deal with description of motion without reference (worry) to the forces that cause them motion
- **dynamics**: relates motion to forces

Fluid Mechanics Terminology

The continuum Hypothesis: we will regard macroscopic behavior of fluids as if the fluids are perfectly continuous in structure. In reality, the matter of a fluid is divided into fluid molecules, and at sufficiently small (molecular and atomic) length scales of fluids cannot be viewed as continuous. However, since we will only consider situations dealing with fluid properties and structure over distances much greater than the average spacing between fluid molecules, we will regard a fluid as a continuous medium whose properties (density, pressure etc.) vary from point to point in a continuous way. For the problems that we will be interested in, the microscopic details of fluid structure will not be needed and the continuum approxim will be appropriate. However, there are situations when molecular level details are important; for instance when the dimensions of a channel that the fluid is flowing through become comparable to the mean free paths of the fluid molecules or to the molecule size. In such instances the continuum hypothesis does not apply.

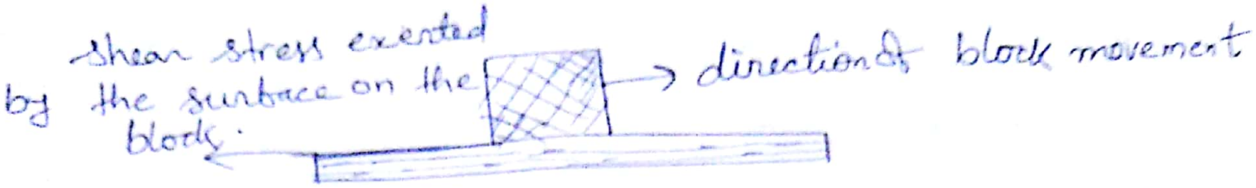
Fluid: a substance that will deform continuously in response to a shear stress no matter how small the stress may be.

Shear stress: Force per unit area that is exerted parallel to the surface on which it acts.

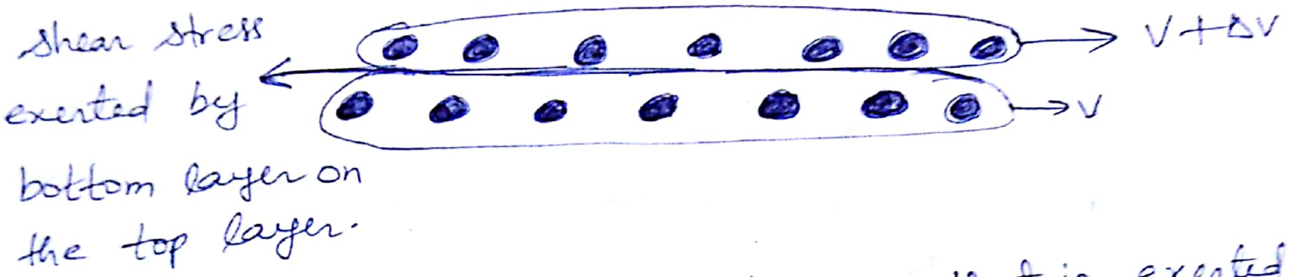
Shear stress units: Force/Area, ex. N/m^2.

usual symbols: σ_{ij}, τ_{ij} $(i \neq j)$.

Example 1: Shear stress between a block and a surface:



Example 2: A simplified picture of the shear stress between two laminas (layers) in a flowing liquid. The top layer moves relative to the bottom one by a velocity ΔV, and collision interactions between the molecules of the two layers give rise to shear stress. Note that the shear stress acts parallel to the surface on which it is exerted.



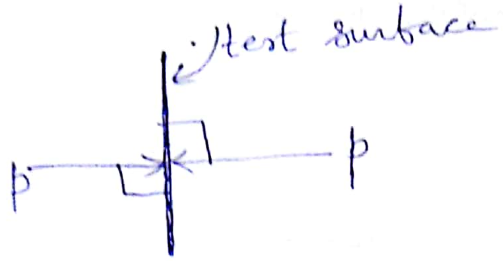
Normal Stress: Force per unit area that is exerted normal to the surface on which it acts. ~~Pressure is a normal stress.~~

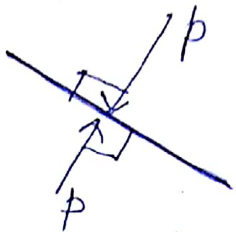
Normal stress units: Force/Area, ex. N/m^2.

usual symbols: σ_{ii}, τ_{ii}.

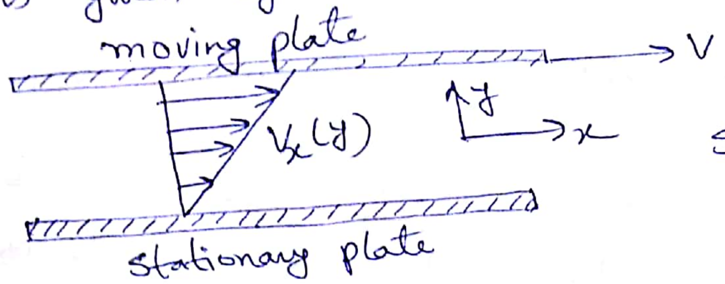
Pressure: A normal, compressive stress that acts on a surface immersed in a fluid. If we have an infinitesimal "test surface" in a fluid, no matter how we orient the test surface, we would measure the same pressure on it as long as the surface is at rest with respect to the fluid around it. Because the pressure does not depend on the orientation of the test surface, we say that it is "isotropic" (i.e. independent of direction). An example of a quantity that is not isotropic is gravitational force, since gravitational force acts along a specific direction.

Pressure units: Force/Area, ex. N/m^2. Usual symbol: p.





Shear Strain Rate (Velocity Gradient): Imagine that we have a velocity $V_x(y)$ as shown below. Then the shear strain rate is given by



Shear Strain Rate $= \frac{dV_x}{dy}$

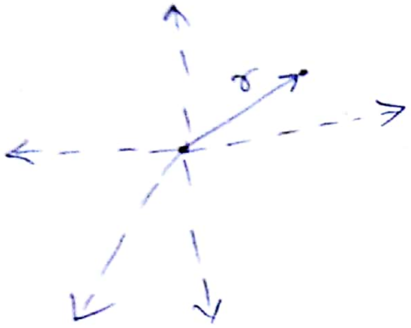
Shear strain rate units: 1/Time, ex. 1/s.

Usual symbols for velocity x-component: V_x, u
y- " : V_y, V
z- " : V_z, w

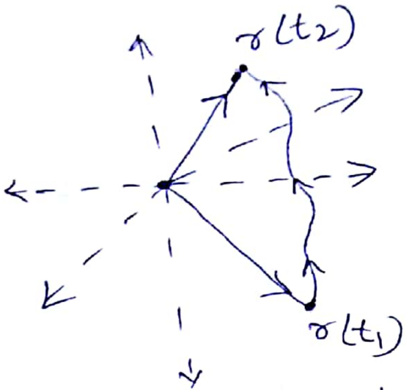
Note that the direction in which the velocity changes (the y direction), is perpendicular to the direction of the velocity (the velocity points along x).

Also, note that usually the velocity of a fluid at solid walls is assumed to be the same as that of the walls (V for the top surface, 0 for the bottom in the figure above). This is known as the "no slip" boundary condition (i.e; the fluid at the fluid/solid surface moves along with the solid surface and does not "slip" relative to the surface).

Mathematical Description of Fluid Flow: Fluid mechanics answers questions such as "How do fluid velocity, density, pressure etc. depend on the position r and time t?" This involves determination of $v(r,t)$, $\rho(r,t)$, $p(r,t)$ etc, where r specifies a point in space with respect to the origin. When r simply refers to a fixed point in space, the problem is said to be formulated in the "Eulerian" representation.



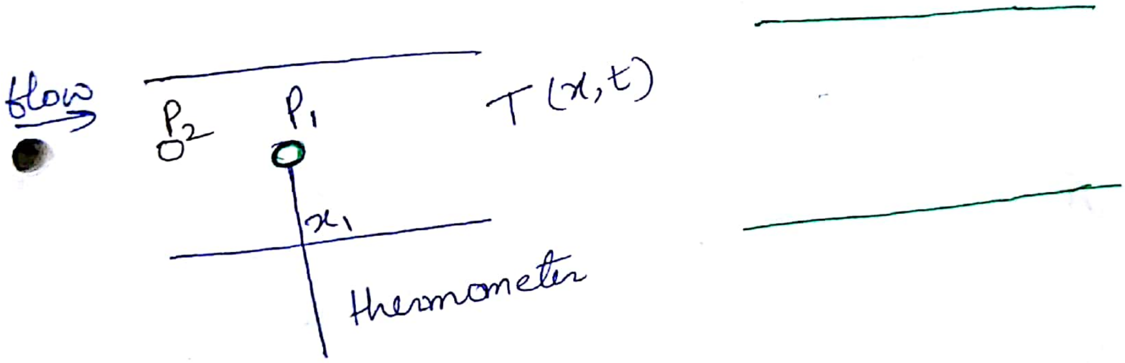
Eulerian representation

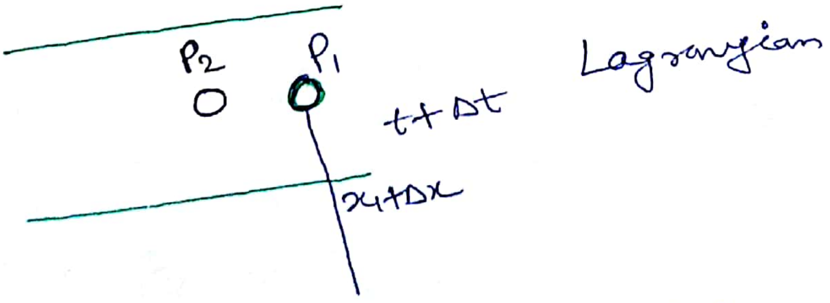


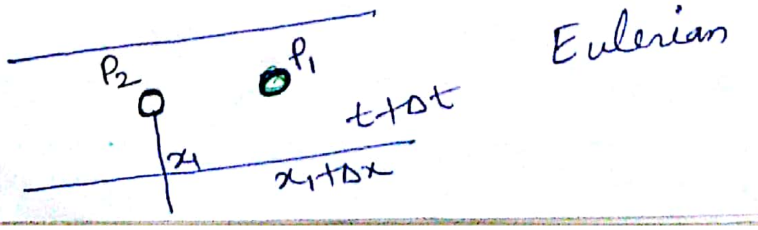
Lagrangian representation

Another representation is "Lagrangian", where $r = r(t)$ is the position of a moving object. Then the velocity of the object is $\frac{dr}{dt}$. In keeping with the most common usage, we will mostly use the Eulerian representation. In summary, when we use r we understand it to simply refer to a position in space (Eulerian representation), and not to the changing position of some object such as a fluid element. Note that in the Eulerian representation, taking derivatives of r with respect to time has no physical meaning.

- Identify various particles by their locations at time t equals to 0 and merely following the positions of various particles as a function of time is called the Lagrangian description.
- following the same fluid particles.
- Material description / Lagrangian description not most useful way.
- Eulerian / spatial description
  - not following ~~same~~ the fluid particles
    - lab frame of reference it could be stationary or move with a constant velocity that depends on the nature of the problem.
    - Put various flow measuring velocity meters at various points in space and then measure velocity at each & every point as a function of time. So that description is the Eulerian description.







To write Newton's second law for an infinitesimal fluid system, we need to calculate the acceleration vector a of the flow. Thus we compute the total time derivative of the velocity vector:

$$a = \frac{dV}{dt} = i\frac{du}{dt} + j\frac{dv}{dt} + K\frac{dw}{dt}$$

Since each scalar component (u, v, w) is a function of the four variables (x, y, z, t), we use the chain rule to obtain each scalar time derivative. For example,

$$\frac{d\,u(x,y,z,t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

But, by definition, dx/dt is the local velocity component u, and dy/dt = v, and dz/dt = w.

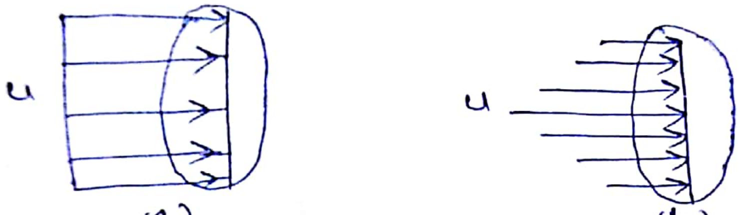
The average velocity is defined as $V_{av} = Q/A$, where $Q = \int u dA$ over the cross-section.

we obtain,

$$V_{av} = \frac{1}{A}\int u dA$$

Velocity: Many problems in fluid mechanics deal with the velocity of the fluid at a point, equal to the rate of change of the position of a fluid particle with time, thus having both a magnitude and a direction.

If the fluid passes through a plane of area A normal to the direction of the velocity, as shown in Figure, the corresponding volumetric flow rate of fluid through the plane is $Q = uA$.



(a) (b)

Fluid passing through an area A:
(a) Uniform velocity, (b) varying velocity

The corresponding mass flow rate is $m = \rho Q = \rho u A$, where ρ is the (constant) fluid density. The alternative notation with an overdot, $\dot{m}$, is also used.

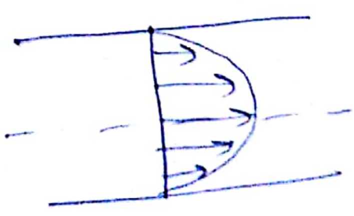
The average velocity u is given by: $u = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}}$.

Flow of viscous fluid through circular pipe

$$\frac{\Delta P}{\mu L} \frac{R^2}{4}\left[1-\frac{r^2}{R^2}\right] = u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} \left[R^2 - r^2\right]$$

$$\frac{\Delta P}{4\mu} \frac{R^2}{L} = u_{max} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$\frac{\Delta P}{8\mu} \frac{R^2}{L} = \overline{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2$$



Velocity Field

Foremost among the properties of a flow is the velocity field $V(x,y,z,t)$. In fact, determining the velocity is often tantamount to solving a flow problem, since other properties ~~to~~ follow directly from the velocity field. Calculation of the pressure field once the velocity field is known. Heat transfer are essentially denoted to finding the temp. field from known velocity fields.

In general, velocity is a vector function of position and time and thus has three components u, v and w, each a scalar field in itself:

$$V(x,y,z,t) = i\,u(x,y,z,t) + j\,v(x,y,z,t) + k\,w(x,y,z,t)$$

Several other quantities, called kinematic properties, can be derived by mathematically manipulating the velocity field. We list some kinematic properties here and give more details about their use and derivation.

1. Displacement vector: $r = \int V\,dt$
2. Acceleration: $a = \frac{dV}{dt}$
3. Volume rate of flow: $Q = \int (V\cdot n)\,dA$
4. Local angular velocity: $\omega = \frac{1}{2}\nabla \times V$

The point of the list is to illustrate the type of vector operations used in fluid mechanics and to make clear the dominance of the velocity field in determining other flow properties. Note: The fluid accl^n, item 2 above, is not as simple as it looks and actually involves four different terms due to use of chain rule in calculus.

Dimensionality of flow

One, Two and Three Dimensional Flows

- Fluid flow is three-dimensional in nature.
- This means that the flow is parameters like velocity, pressure and so on vary in all the three coordinate directions.

Sometimes simplification is made in the analysis of different fluid flow problems by:

- Selecting the appropriate coordinate directions so that appreciable variation of the hydro dynamic parameters take place in only two directions or even in only one.

One-dimensional Flow

- All the flow parameters may be expressed as functions of time and one space coordinate only.
- The single space coordinate is usually the distance measured along the centre-line (not necessarily straight) in which the fluid is flowing
- Example: the flow in a pipe is considered one-dimensional when variations of pressure and velocity occur along the length of the pipe, but any variation over the cross-section is assumed negligible.
- In reality, flow is never one-dimensional because viscosity causes the velocity to decrease to zero at the solid boundaries
- If, however, the non uniformity of the actual flow is not too great, valuable results may often be obtained from a "one dimensional analysis".

- The average values of the flow parameters at any given section (perpendicular to the flow) are assumed to be applied to the entire flow at that section.

Two-dimensional flow

- All the flow parameters are functions of time and two space coordinates (say x and y).
- No variation in z direction.
- The same streamline patterns are found in all planes perpendicular to z direction at any instant.

Three dimensional flow

The hydrodynamic parameters are functions of three space coordinates and time.

Flow Patterns: Streamlines, Streaklines, and Pathlines

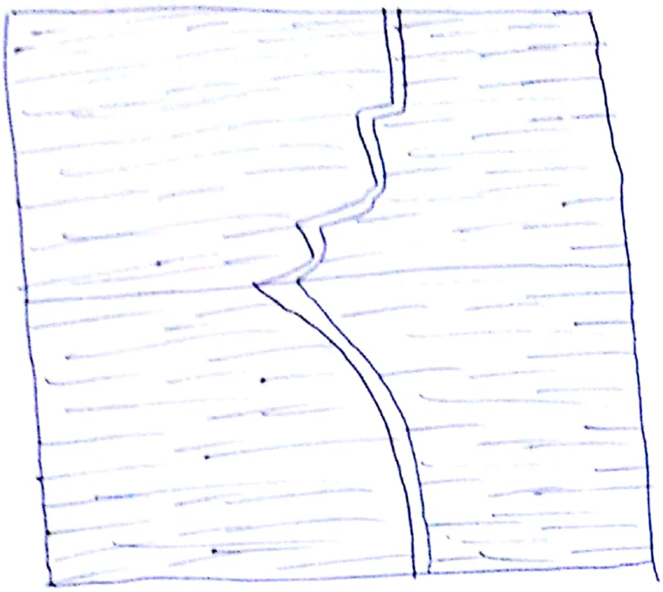
Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways, and you can view these sketches or photographs and learn a great deal qualitatively and often quantitavely about the flow.

Four basic types of line patterns are used to visualize flows:

1. A <u>streamline</u> is a line everywhere tangent to the velocity vector at a given instant.
2. A <u>pathline</u> is the actual path traversed by a given fluid particle.
3. A <u>streakline</u> is the locus of particles which have earlier passed through a prescribed point.
4. A <u>timeline</u> is a set of fluid particles that form a line at a given instant.

The streamline is convenient to calculate mathematically, while the other three are easier to generate experimentally. Note that a streamline and a timeline are instantaneous lines, while the pathline and the streakline are generated by the passage of time. The velocity profile shown in <u>Figure 1</u> is really a <u>timeline</u> generated by a single discharge of bubbles from the wire. A <u>pathline</u> can be found by a time exposure of a single marked particle moving through the flow. <u>Streamlines</u> are difficult to generate experimentally in unsteady flow unless one marks a great many particles and notes their direction of motion during a very short time interval. In steady flow the situation simplifies greatly:

The no slip condition in water flow past a thin fixed plate. The upper flow is turbulent; the lower flow is laminar. The velocity profile is made visible by a line of hydrogen bubbles discharged from the wire across the flow. Figure 1

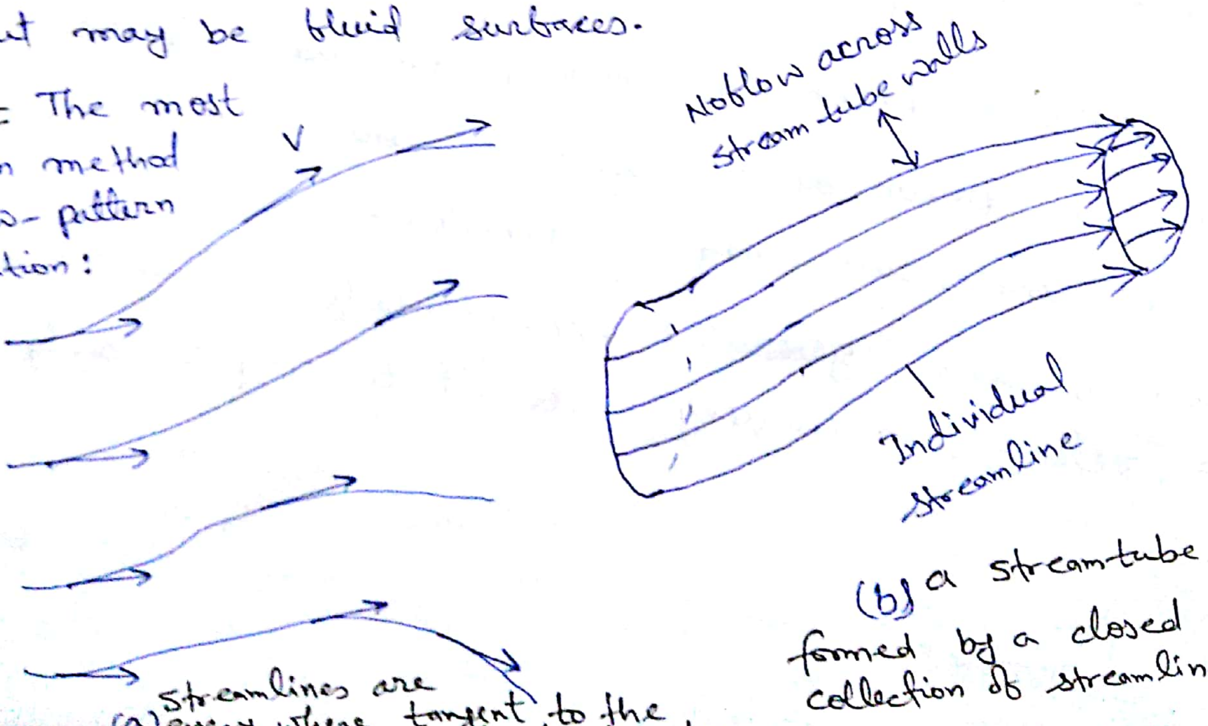


greatly:

- Streamlines, pathlines, and Streaklines are identical in Steady flow.

In fluid mechanics the most common mathematical result for visualization purposes is the streamline pattern. Figure 2(a) shows a typical set of streamlines, and Figure 2(b) shows a closed pattern called a streamtube. By definition the fluid within a streamtube is confined there because it cannot cross the streamlines; thus the streamtube walls need not be solid but may be fluid surfaces.

Figure 2 The most common method of flow-pattern presentation:



(a) streamlines are everywhere tangent to the local vel. vector

(b) a stream-tube is formed by a closed collection of streamlines.

Figure 3 shows an arbitrary velocity vector. If the elemental arc length dr of a streamline is to be parallel to V, their respective components must be in proportion:

Streamline: $$\frac{dx}{u} = \frac{dy}{V} = \frac{dz}{w} = \frac{dr}{V} \quad \text{———} (1)$$

If the velocities (u, v, w) are known functions of position and time, Eqn (1) can be integrated to find the streamline passing through the initial point (x_0, y_0, z_0, t). The method is straight forward for steady flows but may be laborious for unsteady flow.

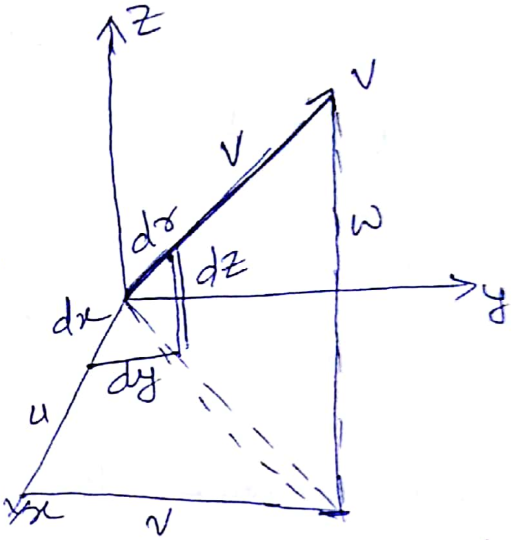


Fig3: Geometric relations for defining a streamline

The pathline, or displacement of a particle, is defined by integration of the velocity components,

Path line: $$x = \int u\,dt, \quad y = \int v\,dt, \quad z = \int w\,dt$$

Given (u, v, w) as known functions of position and time, the integration is begun at a specified initial position (x_0, y_0, z_0, t_0). Again the integration may be laborious.

Streaklines, easily generated experimentally with smoke, dye, or bubble releases, are very difficult to compute analytically.

Prob Given the steady two-dimensional velocity distribution

$$u = Kx \qquad v = -Ky \qquad w = 0 \qquad \text{———} (1)$$

where K is a positive constant, compute and plot the streamlines of the flow, including directions and give some possible interpretations of the pattern.

⇒ **Sol^n:** Since time does not appear explicitly in Eq.(1), the motion is steady, so that streamlines, path lines and streaklines will coincide. Since $w=0$ everywhere, the motion is two dimensional, in the xy plane. The streamlines can be computed by substituting the expressions for u and v into eq^n

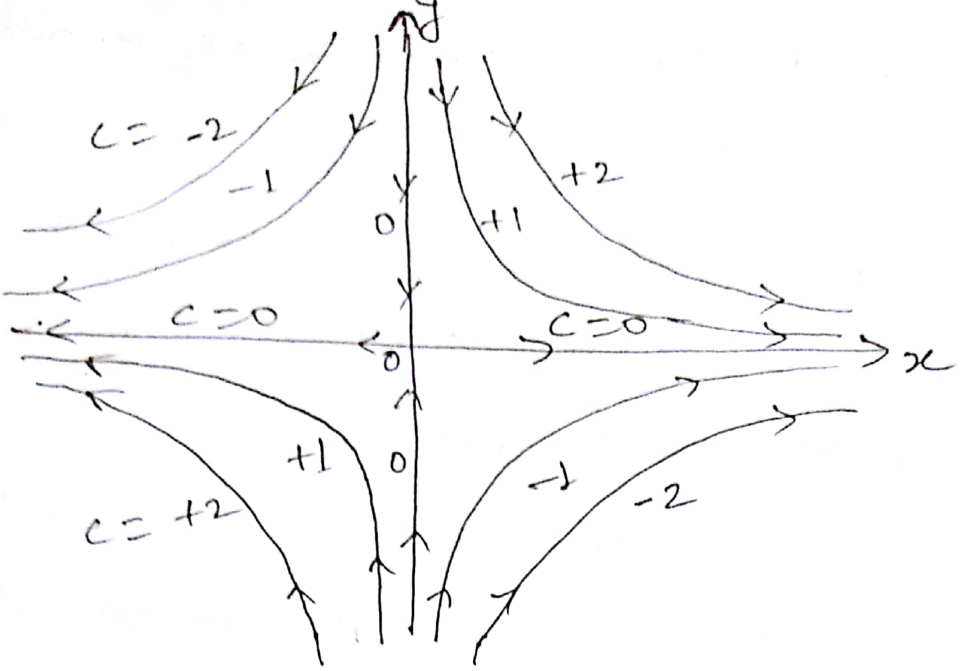
2-D flow $(w=0)$

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{Kx} = \frac{dy}{-Ky} \Rightarrow \int \frac{dx}{x} = -\int \frac{dy}{y}$$

Integrating, we obtain

$$\Rightarrow \ln x = -\ln y + \ln c^{*} \Rightarrow xy = c$$

This is the general expression for the streamlines, which are hyperbolas. The complete pattern is plotted in Figure by assigning various values of the constant c. The arrowheads can be determined only by returning to eq^n (1) to ascertain the velocity component directions, assuming K is positive. For example, in the upper right quadrant $(x>0, y>0)$, u is positive and v is negative; hence the flow moves down and to the right, establishing the arrowheads.



Note that the streamline pattern is entirely independent of constant K. It could represent the impingement of two opposing streams, or the upper half could simulate the flow of a single down ward stream against a flat wall. Taken in isolation, the upper right quadrant is similar to the flow in a 90° corner. This is definitely a realistic flow pattern.

Finally, note ~~that~~ the peculiarity that the two streamlines (C = 0) have opposite directions and intersect. This is possible only at a point where $u = v = w = 0$, which occurs at the origin in this case. Such a point of zero velocity is called a stagnation point.

A streakline can be produced experimentally by the continuous release of marked particles (dye, smoke, or bubbles) from a given point.

Coincidence of lines is always true of steady flow: Since the velocity never changes magnitude or direction at any point, every particle which comes along repeats the behavior of its earlier neighbors.

Substantial derivative

$$\frac{DT}{Dt} = \left.\frac{\partial T}{\partial t}\right)_x + \boxed{\underline{v} \cdot \nabla T} \Rightarrow$$ convected rate of change of Temperature.

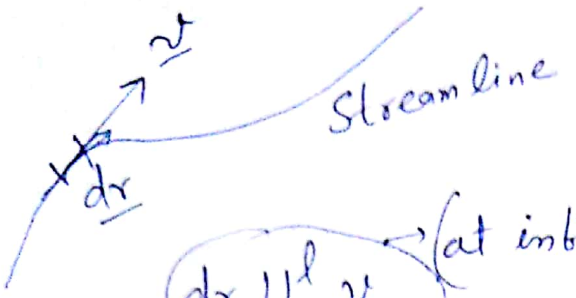
Substantial time derivative of temperature

$\Downarrow$ local rate of change of Temperature

⇒ It allows to calculate the rate of change of many quantities as follow a fluid particle from a given time to a later time purely based on Eulerian description.

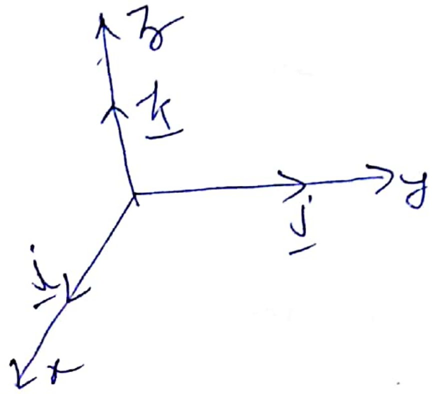
⇒ How temperature will change as you follow particle which is at a given spatial location at given time, this is meaning of the Substantial Derivative.

⇒



$\underline{dr} \parallel^{l} \underline{v}$ ← (at infinitesimal distance on streamline)

$\underline{dr} \times \underline{v} = 0$ (Vector / cross product)



$\underline{dr} = dx\,\underline{i} + dy\,\underline{j} + dz\,\underline{k}$

$\underline{v} = u\,\underline{i} + v\,\underline{j} + w\,\underline{k}$

$\underline{dr} \times \underline{v} = 0$

$$\Rightarrow \begin{vmatrix} \vec{i} & \underline{j} & \underline{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\Rightarrow \underline{i}\,(w\,dy - v\,dz) + \underline{j}\,(u\,dz - w\,dx) + \underline{k}\,(v\,dx - u\,dy) = 0$$

$w\,dy - v\,dz = 0$

$w\,dy = v\,dz$

$\frac{dy}{v} = \frac{dz}{w}$

$v\,dx - u\,dy = 0$

$v\,dx = u\,dy$

$\frac{dx}{u} = \frac{dy}{v}$

$u\,dz - w\,dx = 0$

$u\,dz = w\,dx$

$\frac{dz}{w} = \frac{dx}{u}$.

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

Eqn governing a Streamline

Visiosity

The quantity such as pressure, temperature, and density are primary thermodynamic variables characteristic of any system. There are also certain secondary variables which characterise specific fluid-mechanical behaviour. The most important of these is viscosity, which relates the local stresses in a moving fluid to the strain rate of the fluid element.

When a fluid is sheared, it begins to move at a strain rate inversely proportional to a property called its coefficient of viscosity μ. Consider a fluid element sheared in one plane by a single shear stress τ, as in Figure.

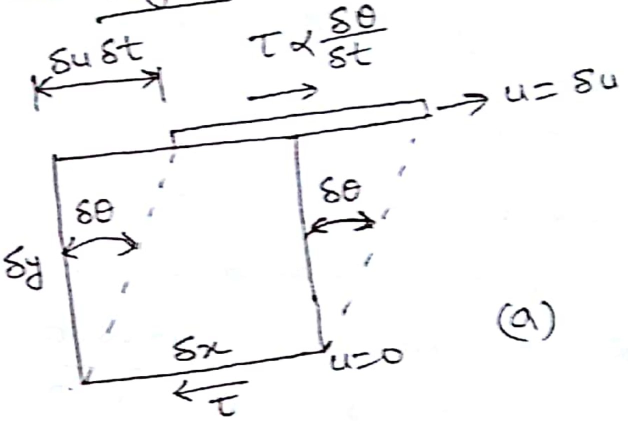


Fig: shear stress causes continuous shear deformation in a fluid: (a) fluid element straining at a rate $\delta\theta/\delta t$;

The shear strain angle $\delta\theta$ will continuously grow with time as long as the stress τ is maintained, the upper surface is moving at speed δu larger than the lower. Such common fluids as water, oil, and air show a linear relation between applied shear and resulting strain rate

$$\tau \propto \frac{\delta\theta}{\delta t} \quad \text{——— (1)}$$

From the geometry of figure we see that

$$\tan \delta\theta = \frac{\delta u\, \delta t}{\delta y}$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient

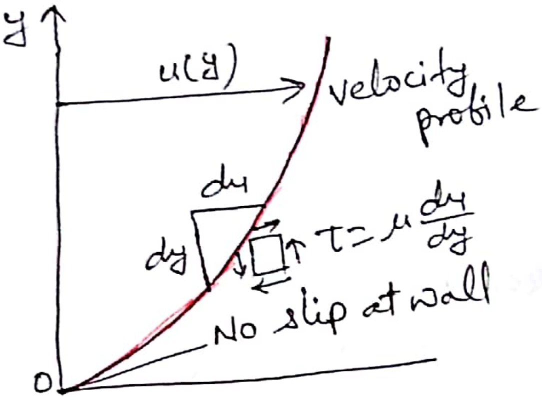
$$\frac{d\theta}{dt} = \frac{du}{dy}$$

From eqn(1), then, the applied shear is also proportional to the velocity gradient for the common linear fluids. The constant of proportionality is the viscosity coefficient μ

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{du}{dy} \quad \text{———} \; (11)$$

Above eqn is dimensionally consistent; therefore μ has dimensions of stress-time: $[FT/L^2]$ or $[M/(LT)]$. The SI unit is kilograms per meter-second. The linear fluids which follow above eqn are called Newtonian fluids, after Sir Isaac Newton, who first postulated this resistance law in 1687.

We do not really care about the strain angle $\theta(t)$ in fluid mechanics, concentrating instead on the velocity distribution $u(y)$. ~~we shall use above eqn~~ Figure(b) illustrates a shear layers or boundary layers near a solid wall. The shear stress is proportional to the slope of the velocity profile and is greatest at the wall. Further, at the wall, the velocity u is 0 zero relative to the wall: This is called the no-slip condition and is characteristic of all viscous-fluid flows.



(b) newtonian shear distribution in a shear layer near a wall.

The viscosity of newtonian fluids is a true thermodynamic property and varies with temp. and pressure. At a given state (P,T) there is a vast range of values among the common fluids. Table1 lists the viscosity of eight fluids at standard pressure and temp. There is a variation of six orders of magnitude from hydrogen upto glycerin. Thus there will be wide differences between fluids subjected to the same applied stresses.

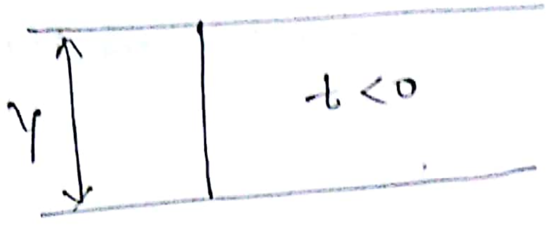
Table 1   Viscosity and Kinematic viscosity of Eight Fluids at 1 atm and 20°c

Fluid	μ kg/(m·s)	ρ kg/m³	ν m²/s
Hydrogen	8.8 E-6	0.084	1.05 E-4
Air	1.8 E-5	1.20	1.51 E-5
Gasoline	2.9 E-4	680	4.22 E-7
Water	1.0E-3	998	1.01 E-6
Ethyl alcohol	1.2 E-3	789	1.52 E-6
Mercury	1.5 E-3	13,580	1.16 E-7
Glycerin	1.5	1,264	1.18 E-3

→ The physical property that characterizes the resistance to flow is the viscosity.

→ Momentum is transferred through the fluid by viscous action

→ Force F is required to maintain the motion of the plate. Common sense suggests that this force may be expressed as follows: $\frac{F}{A} = \mu \frac{V}{Y}$ That is, the force should be proportional to the area and to the velocity, and inversely proportional to the distance b/w the plates. The constant of proportionality μ is a property of the fluid, defined to be the viscosity



Fluid initially
at rest

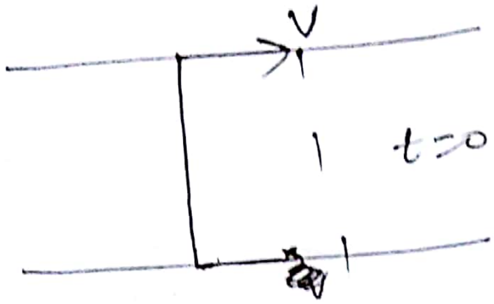
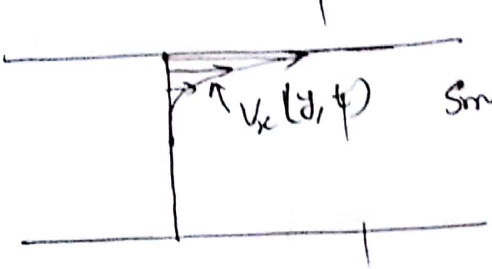
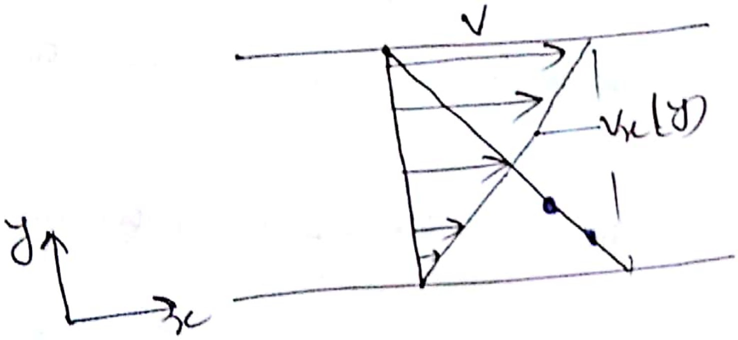


plate set in
motion



Small t

Velocity buildup
in unsteady flow



Large t

Final velocity
distribution in
steady flow.

Generally speaking, the viscosity of fluid increases only weakly with pressure. For example, increasing p from 1 to 50 atm will increase μ of air only 10 percent. Temperature, however, has a strong effect, with μ increasing with T for gases and decreasing for liquids. It is customary in most engineering work to neglect the pressure variation.

~~Newtonian & Non-Newtonian Fluids~~

Variation of Viscosity with Temperature

Temp. has a strong effect and pressure a moderate effect on viscosity. The viscosity of gases and most liquids increases slowly with pressure. Water is anomalous in showing a very slight decrease below 30°C. Since the change in viscosity is only a few percent up to 100 atm, we shall neglect pressure effect.

Gas viscosity increases with temp. Two common approximations are the power law and the Sutherland law:

$$\frac{\mu}{\mu_0} \approx \begin{cases} \left(\frac{T}{T_0}\right)^n & \text{power law} \\ \frac{(T/T_0)^{3/2}(T_0+S)}{T+S} & \text{Sutherland law} \end{cases}$$

where μ_0 is a known viscosity at a known absolute temp. T_0 (usually 273K). The constants n and S are fit to the data, and both formulas are adequate over a wide range of temps.

For air, $n \approx 0.7$ and $S = 110K = 199°R$.

Liquid viscosity decreases with temperature and is roughly exponential, $\mu \approx a e^{-bT}$; but a better fit is the empirical result that $\ln \mu$ is quadratic in $1/T$, where T is absolute temperature.

$$\ln\left(\frac{\mu}{\mu_0}\right) \approx a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2.$$

For water, with $T_0 = 273.16$ K, $\mu_0 = 0.001792$ kg/(m·s), suggested values are $a = -1.94$, $b = -4.80$, and $c = 6.74$, with accuracy about ±1 percent.

Nonnewtonian Fluids

Fluids which do not follow the linear law of eqn (11) are called nonnewtonian. Figure 1(a) compares four examples with a newtonian fluid. A dilatant, or shear-thickening, fluid increases resistance with increasing applied stress. Alternately, a pseudoplastic, or shear-thinning, fluid decreases resistance with increasing stress. The limiting case of a plastic substance is one which requires a finite yield stress before it begins to flow. The linear-flow Bingham plastic idealization is shown, but the flow behaviour after yield may also be nonlinear. An example of a yielding fluid is toothpaste which will not flow out of the tube until a finite stress is applied by squeezing.

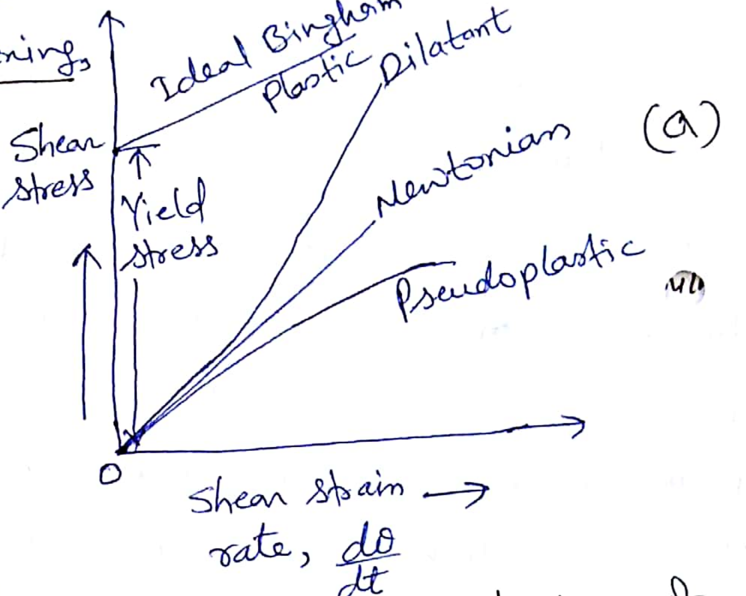
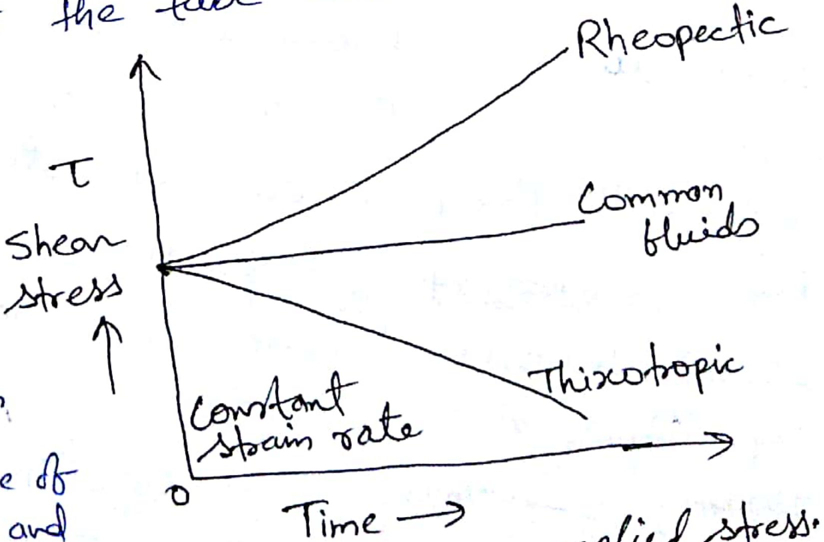
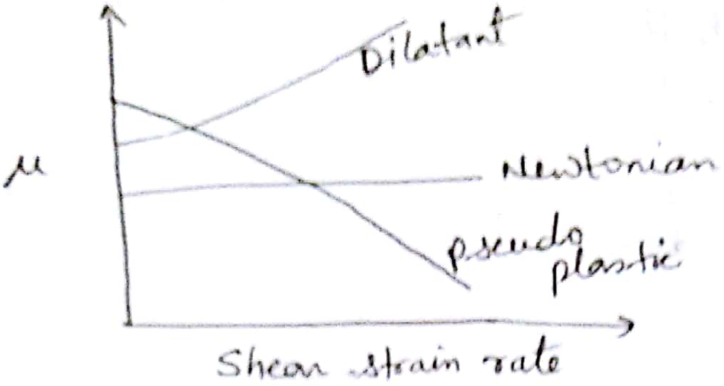


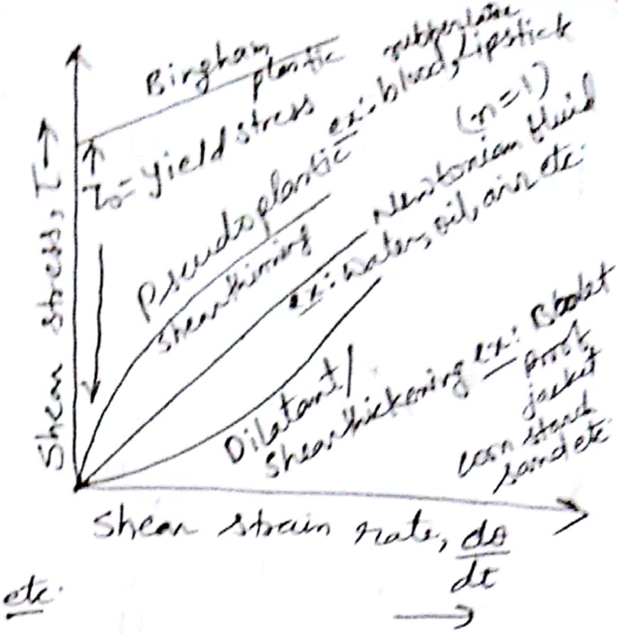
Fig 1. Rheological behaviour of various viscous materials: (a) Stress vs Strain rate

A further complication of nonnewtonian behaviour is the transient effect shown in fig 1(b). Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called rheopectic. The opposite case of a fluid which thins out with time and requires decreasing stress is termed thixotropic.



(b) effect of time on applied stress.





✓ $\tau = \mu \frac{du}{dy}$; Newtonian
ex glycerine etc.

✓ $\tau = m\left(\frac{du}{dy}\right)^n$; power-law

Tomato sauce, Ketchup, rubber latex, milk

If $n<1$, $\tau = m\left(\frac{du}{dy}\right)^n$ shear thinning ex most polymeric system etc.

If $n>1$, shear-thickening ex concentrated soln of sugar in water etc.

$\tau = m\left(\frac{du}{dy}\right)^{n-1} \cdot \frac{du}{dy}$; $m =$ flow consistency index
$n =$ flow behaviour index

$= \mu_a \frac{du}{dy}$

$\mu_a \rightarrow$ apparent viscosity $= m\left(\frac{du}{dy}\right)^{n-1}$

If $n=1$, then $m = \mu$

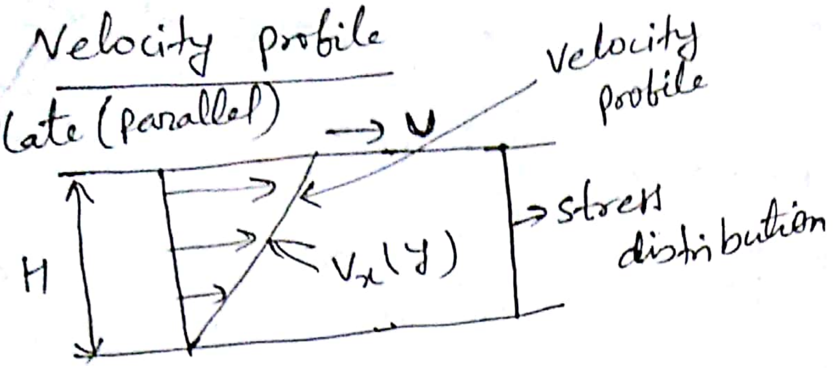
velocity profile ✓ $\tau = \tau_0 + \mu_B \left(\frac{du}{dy}\right)^n$; Bingham plastic

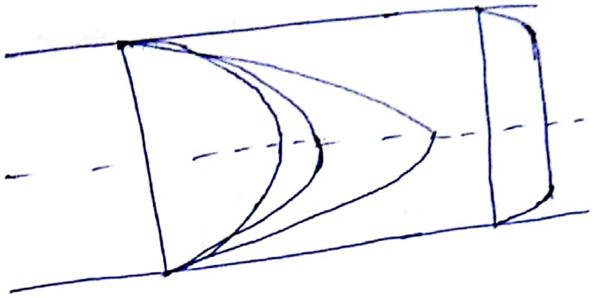
$\mu_B \rightarrow$ plastic viscosity

ex: chocolate, drilling mud etc.
paper pulp.
~~Ketchup,~~ shaving cream

Velocity profile

Plate (parallel)



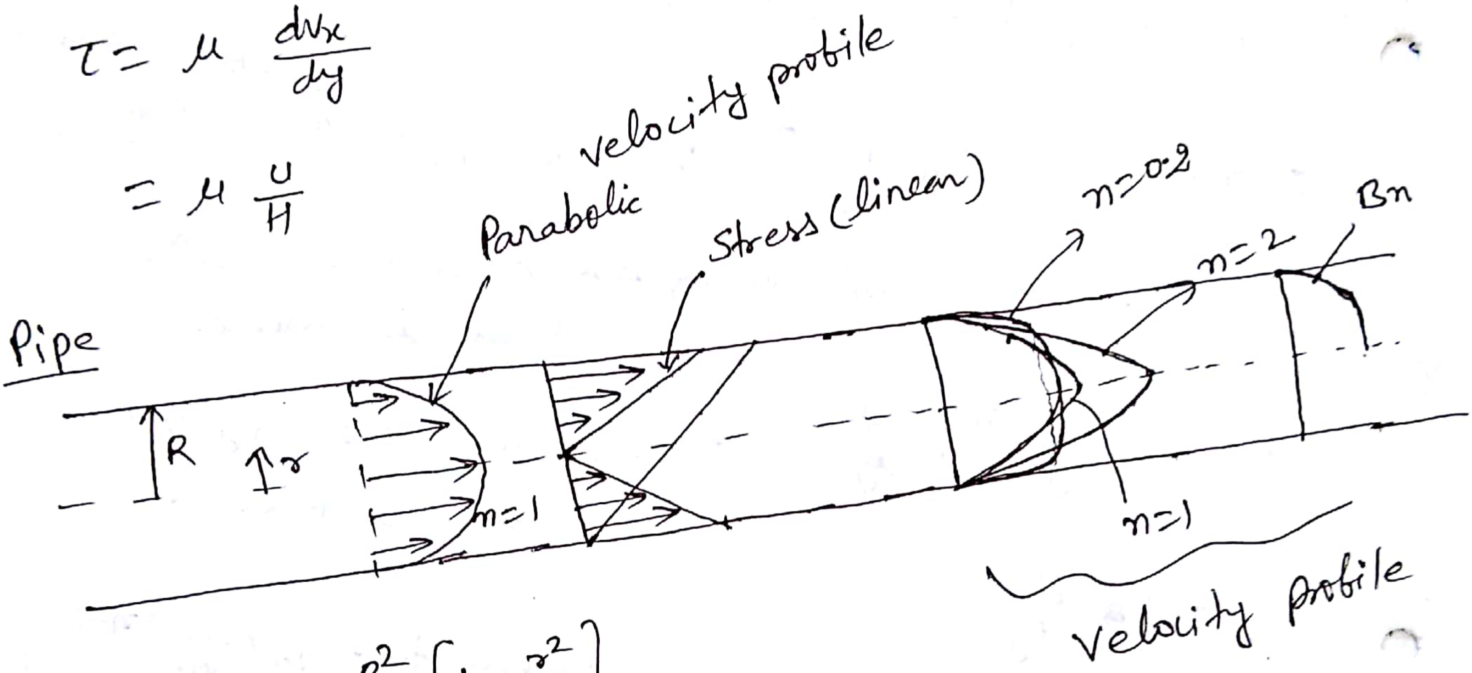


$$V_x(y) = \frac{U}{H} y$$

$$\frac{dV_x(y)}{dy} = \frac{U}{H}$$

$$\tau = \mu \frac{dV_x}{dy}$$

$$= \mu \frac{U}{H}$$

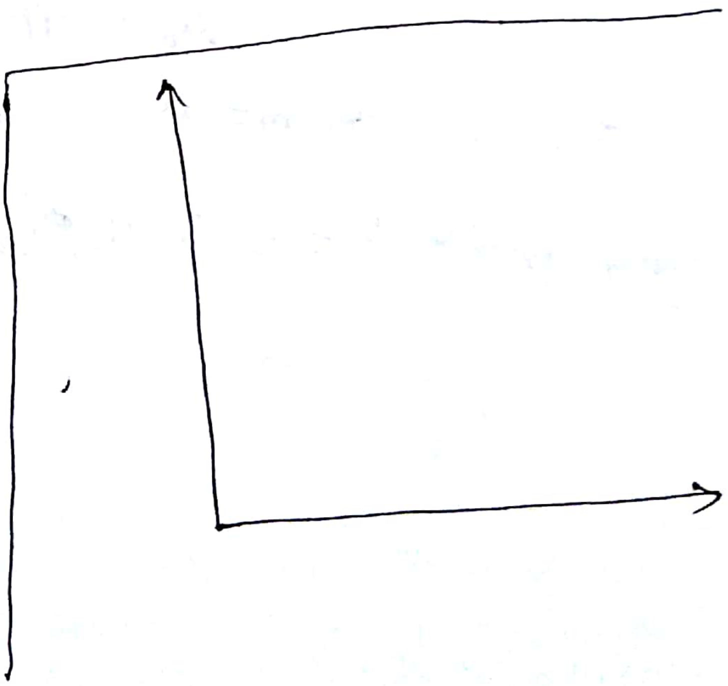


$$u(r) = \frac{\Delta P}{\mu L} \frac{R^2}{4} \left[1 - \frac{r^2}{R^2}\right]$$

$$\frac{du(r)}{dr} = \frac{\Delta P}{\mu L} \frac{R^2}{4} \frac{(-2r)}{R^2}$$

$$= -\frac{\Delta P}{\mu L} \frac{r}{2}$$

$$\tau = \mu \frac{du}{dr} = -\frac{\Delta P}{L} \frac{r}{2}$$



The Reynolds Number

The primary parameter correlating the viscous behaviour of all newtonian fluids is the dimensionless Reynolds number:

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

where V and L are characteristic velocity and length scales of the flow. The second form of Re illustrates that the ratio of μ to ρ has its own name, the kinematic viscosity:

$$\nu = \frac{\mu}{\rho}$$

It is called kinematic because the mass units cancel, leaving only the dimensions $[L^2/T]$.

Generally, the first thing a fluids engineer should do is estimate the Reynolds number range of the flow under study. Very low Re indicates viscous creeping motion, where ~~int~~ inertia effects are negligible. Moderate Re implies a smoothly varying laminar flow,. High Re probably spells turbulent flow, which is slowly varying in the time-mean but has superimposed strong random high-frequency fluctuations. The pecking order changes considerably, and mercury, the heaviest, has the smallest viscosity relative to its own weight. All gases have high ν relative to thin liquids such as gasoline, water and alcohol. Oil and glycerin still have the highest ν, ~~but~~

Laminar Flow

At low velocities fluids tend to flow without lateral mixing, and adjacent layers slide past one another as playing cards do. There are neither cross-currents nor eddies. This regime is called laminar flow. At higher velocities turbulence appears and eddies form, which lead to lateral mixing.

It has long been known that a fluid can flow through a pipe or conduit in two different ways. At low flow rates the pressure drop in the fluid increases directly with the fluid velocity; at high rates it increases much more rapidly, roughly as the square of the velocity. The distinction between the two types of flow was first demonstrated in a classic experiment by Osborne Reynolds, reported in 1833.

A horizontal glass tube was immersed in a glass-walled tank filled with water. A controlled flow of water could be drawn through the tube by opening a valve. The entrance to the tube was flared, and provision was made to introduce a fine filament of colored water from the overhead flask into the stream at the tube entrance. Reynolds found that, at low flow rates, the jet of colored water flowed intact along with the mainstream and no cross-mixing occurred. The behaviour of the color band showed clearly that the water was flowing in parallel straight lines and that the flow was laminar. When the flow rate was increased, a velocity, called the critical velocity, was reached at which the thread of color became wavy and gradually disappeared, as the dye spread uniformly throughout the entire cross section of the stream of water. This behaviour of the colored water showed that the water no longer flowed in laminar motion but moved erratically in the form of cross-currents and eddies. This type of motion is turbulent flow.

Reynolds number and transition from laminar to turbulent flow

Reynolds studied the conditions under which one type of flow changes to the other and found that the critical velocity, at which laminar flow changes to turbulent flow, depends on four quantities: the diameter of the tube and the viscosity, density, and average linear velocity of the liquid. Furthermore, he found that these four factors can be combined into one group and that the change in the kind of flow occurs at a definite value of the group. The grouping of variables so found was

$$Re = \frac{D\bar{V}\rho}{\mu} = \frac{D\bar{V}}{\nu}$$

where, D = diameter of tube

$\bar{V}$ = average velocity of liq.

$$= \frac{\dot{m}}{\rho A} = \frac{1}{A}\int_A u\,dA$$

$$\dot{m} = \rho \int_A u\,dA$$

$$\bar{V} = \frac{q}{A}$$

$q \rightarrow$ volumetric flow rate

μ = viscosity of liquid

ρ = density of liquid

ν = kinematic viscosity of liquid.

The dimensionless group of variables defined above is called the Reynolds number Re. Its magnitude is independent of the units used. Additional observations have shown that the transition from laminar to turbulent flow actually may occur over a wide range of Reynolds numbers. In a pipe, flow is always laminar at Reynolds numbers below 2100, but laminar flow can persist in smooth tubes up to Reynolds numbers well above 2100 by eliminating all disturbances at the inlet. If the laminar flow at such high Reynolds numbers is disturbed, however, say by a fluctuation in velocity, the flow quickly becomes turbulent. Disturbances under these conditions are amplified, whereas at Reynolds numbers below 2100 all disturbances are damped and the flow remains laminar. At some flow rates a disturbance may be neither damped nor amplified; the flow is then said to be neutrally stable. Under ordinary conditions, the flow in a pipe or tube is turbulent at Reynolds numbers above about 4,000. Between 2100 and 4000 a transition region is found where the flow may be either laminar or turbulent, depending upon conditions at the entrance of the tube and on the distance from the entrance.

$$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{ma}{\mu \frac{V}{y} A} = \frac{\overset{tQ}{\cancel{\forall}}\, \rho a}{\mu A \frac{V}{y}} = \frac{\cancel{t}\cancel{A} V \rho \frac{V}{\cancel{t}}}{\mu \cancel{A} \frac{\cancel{V}}{y}} = \frac{\rho V y}{\mu}$$

pipe $\frac{\rho D V}{\mu}$

Boundary Layers

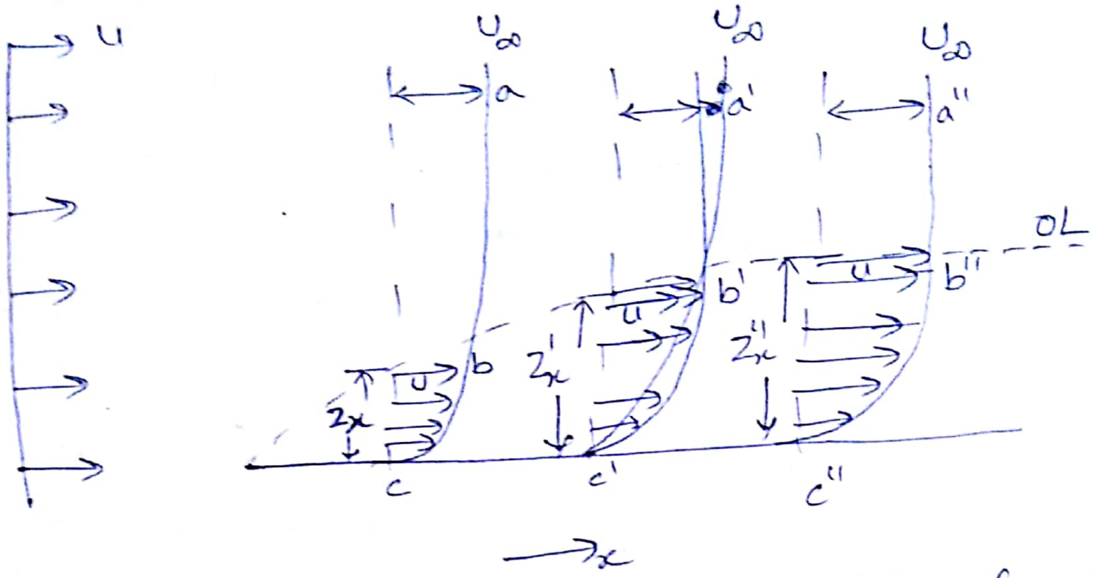


Figure Prandtl boundary layer: x, distance from leading edge; u_∞, velocity of undisturbed stream; Z_x, thickness of boundary layer at distance x; u, local velocity; abc, a'b'c', a''b''c'', curves of velocity versus distance from wall at points c, c', c''; OL, outer limit of boundary layer

Flow in boundary layers

A boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presence of a solid boundary. As a specific example of boundary layer formation, consider the flow of fluid parallel with a thin plate. The velocity of the fluid upstream from the leading edge of the plate is uniform across the entire fluid stream. The velocity of the fluid at the interface between the solid and fluid is zero. The velocity increases with distance from the plate. Each of these curves corresponds to a definite value of x, the distance from the leading edge of the plate. The curves change slope rapidly near the plate; they also show that the local velocity approaches asymptotically the velocity of the bulk of the fluid stream.

The dashed line OL is so drawn that the velocity changes are confined between this line and the trace of the wall. Because the velocity lines are asymptotic with respect to distance from the plate, it is assumed, in order to locate the dashed line definitely, that the line passes through all points where the velocity is 99 percent of the bulk fluid velocity u_∞. Line OL represents an imaginary surface that separates the fluid stream into two parts: one in which the fluid velocity is constant and the other in which the velocity varies from zero at the wall to a velocity substantially equal to that of the undisturbed fluid.

This imaginary surface separates the fluid that is directly affected by the plate from that in which the local velocity is constant and equal to the initial velocity of the approach fluid. The zone, or layer, between the dashed line and the plate constitutes the boundary layer.

The formation of and behaviour of the boundary layer are important, not only in the flow of fluids but also in the transfer of heat and mass.

Laminar and turbulent flow in boundary layers

The fluid velocity at the solid - fluid interface is zero, and the velocities close to the solid surface are, of necessity, small. Flow in this part of the boundary layer very near the surface therefore is essentially laminar. Actually it is laminar most of the time, but occasionally eddies from the main portion of the flow or the outer region of the boundary layer move very close to the wall, temporarily disrupting the velocity profile. These eddies may have little effect on the average velocity profile near the wall, but they can have a large effect on the profiles of temp or concentration when heat or mass is being transferred to or from the wall. This effect is most pronounced for mass transfer in liquids.

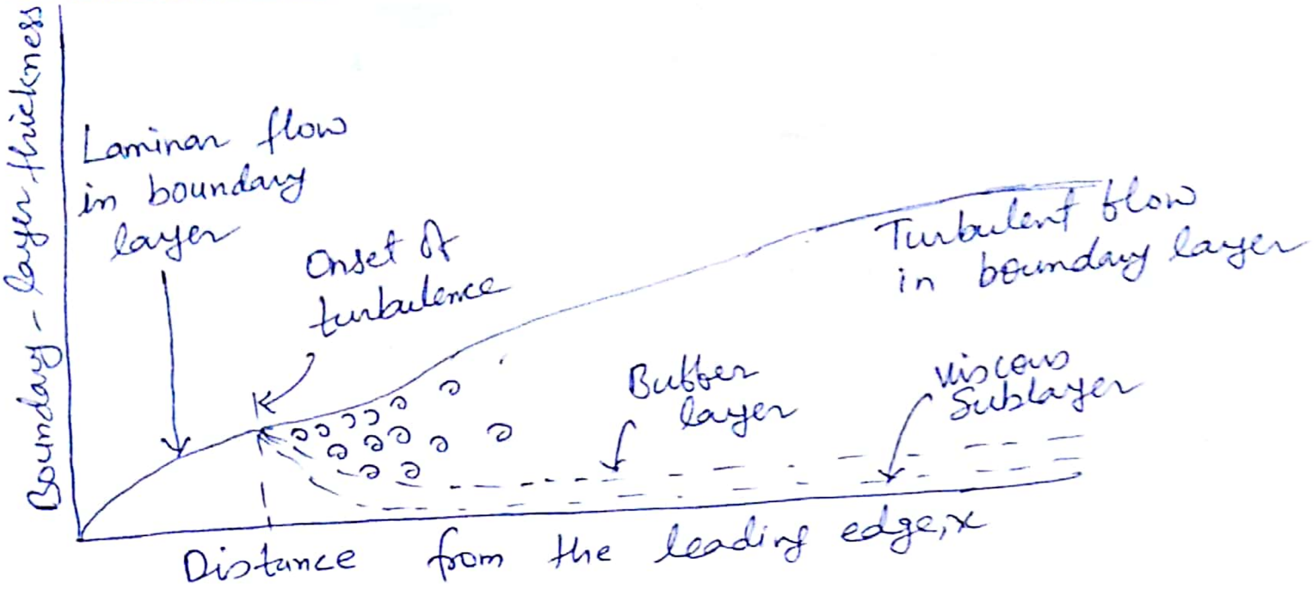
Farther away from the surface the fluid velocities, though less than the velocity of the undisturbed fluid, may be fairly large, and flow in this part of the boundary layer may become turbulent. Between the zone of fully developed turbulence and the region of laminar flow is a transition, or buffer, layer of intermediate character.

Thus a turbulent boundary layer is considered to consist of three zones: the viscous sublayer, the buffer layer, and the turbulent zone.

Near the leading edge of a flat plate immersed in a fluid of uniform velocity, the boundary layer is thin, and the flow in the boundary layer is entirely laminar. As the layer thickenes, however, at distances farther from the leading edge, a point is reached where turbulence appears. The onset of turbulence is characterized by a sudden rapid increase in the thickness of the boundary layer.

When the flow in the boundary layer is laminar, the thickness Z_x of the layer increases with $x^{0.5}$, where x is the distance from the leading edge of the plate. For a short time after turbulence appears, Z_x, increases with $x^{1.5}$ and then, after turbulence is fully developed, with $x^{0.8}$

The initial, fully laminar part of the boundary layer may grow to a moderate thickness of perhaps 2mm



2mm with air or water moving at moderate velocities.

Once ~~tub~~ turbulence begins, however, the thickness of the laminar part of the boundary layer diminishes considerably, typically to about 0.2 mm.

Incompressible Flow

Fluid is incompressible, density variation which are negligeble. All liquids are nearly incompressible, and gas flows can behave as if they were incompressible, particularly if the gas velocity is less than about 30% of the speed of sound of the gas.

Compressible Flow

When a fluid moves at speeds comparable to its speed of sound, density changes become significant and the flow is termed compressible. Such flows are difficult to obtain in liquids, since high pressures of order 1000 atm are needed to generate sonic velocities. In gases, however, a pressure ratio of only 2:1 will likely cause sonic flow. Probably the two most important and distinctive effects of compressibility on flow are (1) choking, wherein the duct flow rate is sharply limited by the sonic condition, and (2) shock waves, which are nearly discontinuous property changes in a supersonic flow.

The proper criterion for a nearly incompressible flow was a small Mach number $Ma = \frac{V}{a} << 1$.

where V is the flow velocity and a is the speed of sound of the fluid.

Under small-Mach number conditions, changes in fluid density are everywhere small in the flow field. The energy eqn becomes uncoupled, and temp. effects can be ignored. The eqn of state degenerates into the simple statement that density is nearly constant.

Newton's Law of Viscosity Imp.

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-Newtonian fluids.

Liq, $\mu \downarrow$ $T \uparrow$

gas, $\mu \uparrow$ $T \uparrow$

intermolecular force of attraction (cohesive force) & molecular momentum transfer.

Due to closely packed molecules in liquid, intermolecular force of attaraction predominates the molecular momentum transfer and with ~~inde~~ increase in temp., the intermolecular force of attaraction decreases with the result of decreasing viscosity

But in the case of gases the cohesive force are Small and molecular momentum transfer predominates. With the increase in temp., molecular momentum transfer increases and hence viscosity increases.

The relation b/w viscosity and temp. for liquids and gases are :

Viscosity of liq at 0°c

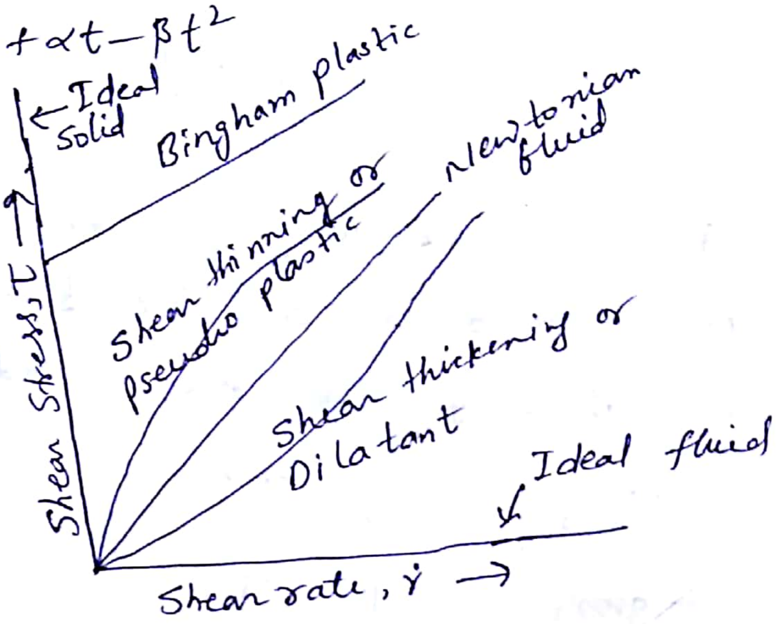
For liquid, $\mu = \mu_0 \left(\frac{1}{1+\alpha t+\beta t^2} \right)$

Viscosity of liq at t°c

α, β = constants for liquid

For gas, $\mu = \mu_0 + \alpha t - \beta t^2$

Type of fluids



1. Ideal fluid

A fluid which is in compressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Read Fluid

A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

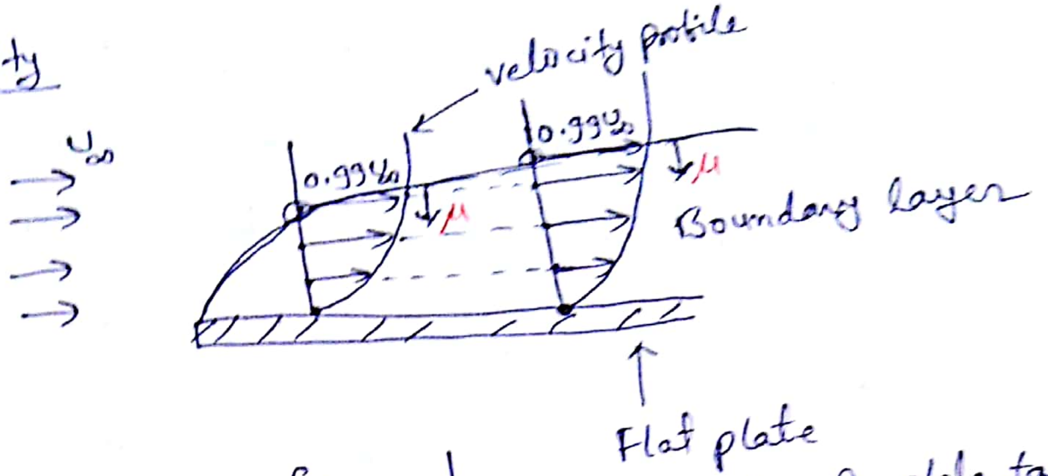
3. Newtonian Fluid

A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid

A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity

Viscosity



→ Fluid is coming from far stream with uniform velocity u_∞ | (just like top of ~~plate~~ table)

∞ subscript indicates normaly for free stream velocity

→ Now flow is disturbed because of the presence of the plate.

→ When this fluid 1st comes in contact with the plate what happen

→ Let 1st fluid molecule which comes in contact with plate

→ Gas molecule adsorbed on the surface

It will exchange some of its momentum with the surface

It will try to have slow down

geting ejected from the surface

In this process many molecules are colliding this and exchangeng their momentum with the wall

very large n. of collisions . theoretically infinitely

n. of collisions

Then this type of momentum exchange on an average eq^lbm with the surface so If the surface is at rest the molecules will be also at rest

That will imply that zero relative velocity at point of contact this is somthing which is known as NO-Slip boundary conditions

No-slip boundary condition
It is zero relative tangential component of velocity b/w fluid and the solid at their points of contact.

No Slip BC only talk about tangential velocity component

→ Shear stress is proportional to rate of shear strain
A class of fluids which obey this rule is called Newtonian.

Dynamic Viscosity
If a fluid with a viscosity of 1 Pa·s is placed between two plates, and one plate is pushed sideways with a shear stress of 1 pascal, it moves a distance equal to the thickness of the layer b/w the plates in one second.

$\eta = \dfrac{\text{amount of an extensive quantity}}{\text{per unit mass}}$ $\begin{cases} \text{— mass} \\ \text{— momentum} \\ \text{— energy} \end{cases}$

$N = m$ mass $1 =$

$N = mv$ momentum $\underline{v} =$ $\quad \eta = \frac{N}{m}$

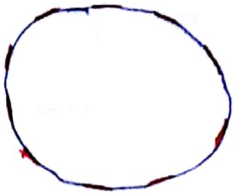
$N = \frac{1}{2}mv^2$ kinetic energy $\frac{1}{2}v^2 =$

Reynolds Transport theorem

$N =$ total amount of mass $= m$ } Present in system

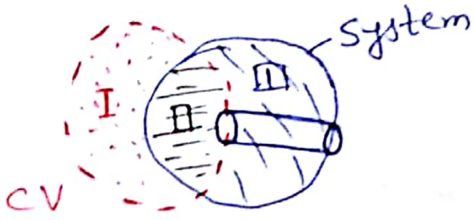
momentum $= m\underline{v}$

energy $= \frac{1}{2}m\,\underline{v}\cdot\underline{v}$



$t = t_0$

System + CV coincide



$t = t_0 + \Delta t$

control volume

fixed region in space

System

control mass

identifiable piece of matter

$$N_{sys} = \int_{\forall_{sys}} \eta \rho \, d\forall$$

$$\frac{dN_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{N_{sys}\big)_{t_0+\Delta t} - N_{sys}\big)_{t_0}}{\Delta t}$$

$$N_{sys}\Big|_{t_0+\Delta t} = \left(N_{II} + N_{III}\right)_{t_0+\Delta t} = \left[N_{CV} - N_{I} + N_{III}\right]_{t_0+\Delta t}$$

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$$N_{sys}\Big)_{t=t_0} = N_{cv}\Big)_{t_0}$$ because the system & CV coincide at t_0

$$\frac{dN_{sys}}{dt} = \lim_{\Delta t \to 0} \frac{\left[N_{cv} - N_I + N_{III}\right]_{t_0+\Delta t} - \left[N_{cv}\right]_{t_0}}{\Delta t}$$

$$= \underbrace{\boxed{\lim_{\Delta t \to 0} \frac{N_{cv}\Big)_{t_0+\Delta t} - N_{cv}\Big)_{t_0}}{\Delta t}}}_{\frac{\partial N_{cv}}{\partial t} \Rightarrow \frac{\partial}{\partial t}\int_{C.V.} \eta \rho \, dV} + \lim_{\Delta t \to 0} \frac{N_{III}\Big)_{t_0+\Delta t}}{\Delta t} - \lim_{\Delta t \to 0} \frac{N_I\Big)_{t_0+\Delta t}}{\Delta t}$$

RTT is a tool to convert time derivative of system in terms of time derivative of the variables that are present in C.V.

This is very similar to the "conversion of a Eulerian time derivative to the Lagrangian time derivative by virtue of the substantial derivative" that only that we are not taking about a single point in space or a single material point but here a microscopic volume called control volume.

⇒ RTT is a way of relating time derivatives of quantity such as mass in a given region of space to that with associated with the system or the material that is present in the system.

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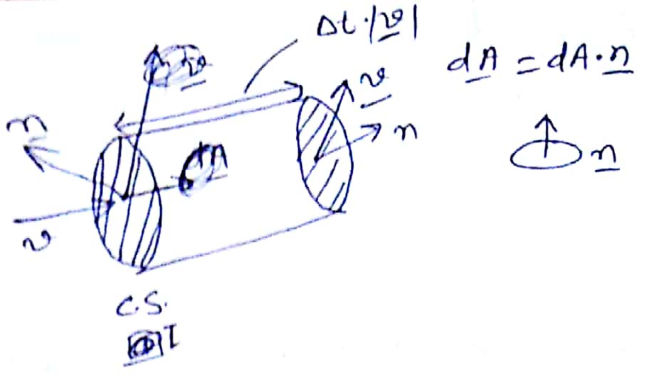
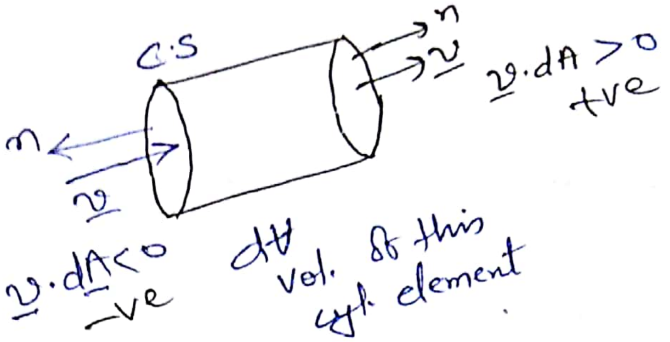
$$\lim_{\Delta t\to 0} \frac{N_{III})_{t_0+\Delta t}}{\Delta t} \quad ?$$



$$dN_{III})_{t_0+\Delta t} = (\eta\rho\, d\forall)_{t_0+\Delta t}$$

$$d\forall = (\underline{v}\cdot d\underline{A})\,\Delta t$$



$$dN_{III})_{t_0+\Delta t} = \eta\rho\, d\forall)_{t_0+\Delta t}$$

$$= \eta\rho\,(\underline{v}\cdot d\underline{A})\,\Delta t$$

$$\lim_{\Delta t\to 0} \frac{N_{III})_{t_0+\Delta t}}{\Delta t} = \lim_{\Delta t\to 0} \frac{1}{\Delta t} \int_{\substack{C.S.\\ III}} dN_{III})_{t_0+\Delta t}$$

$$= \lim_{\Delta t\to 0} \frac{1}{\cancel{\Delta t}} \int_{\substack{C.S.\\ III}} \eta\rho\,(\underline{v}\cdot d\underline{A})\,\cancel{\Delta t}$$

$$\lim_{\Delta t\to 0} \frac{N_{III})_{t_0+\Delta t}}{\Delta t} = \int_{\substack{C.S.\\ III}} \eta\rho\,\underbrace{(\underline{v}\cdot d\underline{A})} \rightarrow \text{Volumetric flow rate}$$

$$\lim_{\Delta t\to 0} \frac{N_{I})_{t_0+\Delta t}}{\Delta t} = -\int_{CS\,I} \eta\rho\; \underline{v}\cdot d\underline{A}$$

$$\left.\frac{dN_{sys}}{dt}\right)_{t_0} = \frac{\partial}{\partial t}\int_{C.V.} \eta\rho\, d\forall + \underbrace{\int_{CS\,I} \eta\rho\; \underline{v}\cdot d\underline{A} + \int_{CS\,III} \eta\rho\; \underline{v}\cdot d\underline{A}}_{\int_{C.S.} \eta\rho\, \underline{v}\cdot d\underline{A}}$$

Reynolds Transports Theorem

$$\frac{dN_{sys}}{dt} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \underline{v} \cdot d\underline{A}$$

this is similar to the connected contribution to fin the substantial deriving → Surface contribution

rate of change of quantity like mass, momentum or energy } extensive qty that present in the system at a given time ~~to~~ to ↑ system & CV coincide

⇓ bulk rate of change of quantities in the CV itself

local rate of change present in CV.

motion of fluid in & out of CV due to fluid flow that is flux term.

flux of qty. moving in/out of the C.V. through C.S.

Macroscopic Balances: are concerned with entire equipment, process equipment like pumps, compressors or tubes, network of tubes & so on.

Integral Balances because the eqns in the form of integral form.

→ cannot give detailed information like what is velocity at each & every point in space, what is shear stress at a point in a wall & so on

R.T.T.

$$\frac{d}{dt}\int_{\forall_{sys}} \eta \rho \, d\forall = \frac{\partial}{\partial t}\int_{C.V.} \eta \rho \, d\forall + \int_{C.S.} \eta \rho \, \underline{v} \cdot \underline{n} \, dA$$

$$\eta = \frac{Mom}{mass} = \frac{m\underline{v}}{m} = \underline{v}$$

$$\underline{F} = \underline{F}_{body} + \underline{F}_s$$

F_s is usually due to P_r, viscous stresses although they are imp. in many cases at level of integral balances. It is not easy to obtain detail information about the viscous shear stresses in a flow problem and often neglect it because of lack of information.

$$\frac{d}{dt}\int_{\forall_{sys}} \rho \underline{v} \cdot d\forall = \frac{\partial}{\partial t}\int_{CV} \rho \underline{v} \, d\forall + \int_{C.S.} \rho \underline{v} \; \underline{v} \cdot \underline{n} \, dA$$

$$\frac{\partial}{\partial t}\int_{CV} \rho \underline{v}\, dV + \int_{C\cdot S} \rho \underline{v}\, \underline{v}\cdot n\, dA = \left(\underline{F}_s + \underline{F}_B\right)$$

→ forces acting on CV

$\underline{F}_s$ ← due to pressure $= \int_{CS} p(-\underline{n})\,dA$

Integral momentum balances

$$\boxed{\underline{F}_s + \underline{F}_B = \frac{\partial}{\partial t}\int_{CV} \rho \underline{v}\, dV + \int_{CS} \rho \underline{v}\, \underline{v}\cdot \underline{n}\, dA}$$

→ surface/area integral

$\frac{\partial}{\partial t}\int_{CV} \rho \underline{v}\, dV$ ↓ how momentum changes in fixed region of space i.e. control volume

$\int_{CS} \rho \underline{v}\, \underline{v}\cdot \underline{n}\, dA$ ↓ momentum flux term

→ how momentum enter or exits control volume by virtue of flow.

$F_B =$ gravity $= \int_{CV} \rho \underline{g}\, dV$

CV → ~~Viscous~~ shear stress tangential

↑ Normal (Pressure)

$\frac{kg}{m^3}\ \frac{m}{s}\ \frac{m}{s}\ m^2 \left(\frac{mom}{time}\right)$

$\frac{kg\, m}{s^2}$

Momentum - Flux Corrector Factor

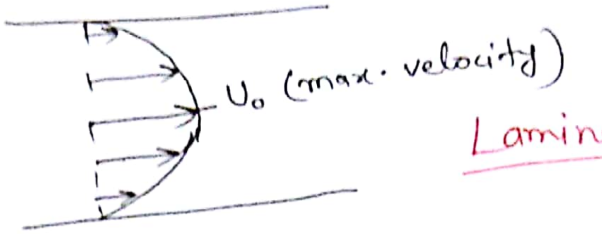
For flow in a circular cylinder, the axial velocity is usually non uniform. For this case the simple momentum-flux calculation $\int u \rho (\underline{v} \cdot \underline{n})\, dA = \dot{m} V = \rho A V^2$ is some what in error and should be corrected to $\beta \rho A V^2$, where β is the dimensionless momentum flux correction factor, $\beta \geq 1$.

The factor β accounts for the variation of u^2 across the circular section. That is, we compute the exact flux and set it equal to a flux based on average velocity in the cross section.

$$\rho \int u^2 dA = \beta \dot{m} V_{av} = \beta \rho A V_{av}^2$$

$V_{av} \equiv \frac{1}{A}\int u\, dA$;

Laminar flow in pipe; $u = U_0\left(1-\frac{r^2}{R^2}\right)$



Laminar

$$V_{av} \equiv \frac{1}{A}\int_{\theta=0}^{2\pi}\int_{r=0}^{R} U_0\left(1-\frac{r^2}{R^2}\right) r\,dr\,d\theta$$

$$= \frac{2\pi U_0}{A}\int_{r=0}^{R}\left(1-\frac{r^2}{R^2}\right) r\,dr = \frac{2\pi U_0}{A}\int_{r=0}^{R}\left[r-\frac{r^3}{R^2}\right]dr$$

$$= \frac{2\pi U_0}{A}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = \frac{2\pi U_0}{A}\left[\frac{R^2}{2}-\frac{R^4}{4R^2}\right]$$

$$V_{av} = \frac{2\pi U_0}{A}\left[\frac{R^2}{4}\right] = \frac{\pi R^2 U_0}{2A} ; \qquad A = \pi R^2$$

$$\boxed{V_{av} = \frac{U_0}{2}}$$

momentum correction factor

V_{av} = c.s. average velocity

$$\int \rho u^2\, dA \equiv \textcircled{\beta}\, \rho A V_{av}^2$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{R} \rho u^2 r\,dr\,d\theta = \beta \rho V_{av}^2 A$$

$$= \beta \rho \frac{\pi R^2 U_0^2}{4}$$

$$2\pi\rho\int_{r=0}^{R} U_0^2 \left(1-\frac{r^2}{R^2}\right)^2 r\,dr = 2\pi\rho U_0^2 \int_{r=0}^{R}\left[1-\frac{2r^2}{R^2}+\frac{r^4}{R^4}\right] r\,dr$$

$$= 2\pi\rho U_0^2 \int_{r=0}^{R}\left[r-\frac{2r^3}{R^2}+\frac{r^5}{R^4}\right]dr$$

$$= 2\pi\rho u_0^2\left[\frac{r^2}{2} - \frac{2r^4}{4R^2} + \frac{r^6}{6R^4}\right]_{r=0}^{R}$$

$$\Rightarrow 2\pi\rho \cancel{u_0^2}\left[\cancel{\frac{R^2}{2}} - \cancel{\frac{R^2}{2}} + \frac{R^2}{6}\right] = \beta\rho\frac{\pi R^2 \cancel{u_0^2}}{4}$$

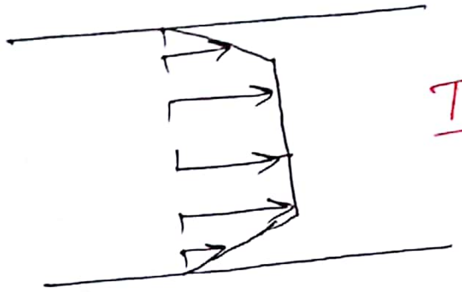
$$2\pi\rho\frac{R^2}{6} = \beta\rho\frac{\pi R^2}{4} \Rightarrow \beta = \frac{8}{6} = \frac{4}{3}$$

$\boxed{\beta = \frac{4}{3}}$ For laminar flow in pipe

Turbulent flow

$u \simeq U_0\left(1 - \frac{r}{R}\right)^{1/r}$ (experimental fact)

$$V_{av} = \frac{1}{A}\int_{\theta=0}^{2\pi}\int_{r=0}^{R} u\,r\,dr\,d\theta$$



Turbulent

$V_{av} = \frac{49}{60} U_0$ (Turbulent)

$V_{av} = \frac{1}{2} U_0 = \frac{30}{60} U_0$ (Laminar)

Similar process as above (excercise)

$\boxed{\beta = 1.020}$ Turbulent flow

→ β more close!

So, variation in turbulent is much more than the laminar because avg vel. is half of max. vel. while in turbulent flow avg vel. is pretty much close to max. vel. because flow is already uniform in core of pipe except very close to the walls, where vel. is varying rapidally zero.

Fluid Flow phenomena: Fluid as continuum, Terminologies of fluid flow, velocity – local, averge, maximum, flow rate – mass, volumetric, velocity field; dimensionality of flow; flow visualization; Streamline, path line, Streakline, stress field, viscosity, Newtonian / Non Newtonian fluid, Reynolds Number – its significance, laminar, transition & turbulent flow; Prandtl Boundary layer, compressible and incompressible, momentum eqn for integral control volume, momentum correction factor